Sumerianz Journal of Education, Linguistics and Literature, 2019, Vol. 2, No. 9, pp. 73-76
ISSN(e): 2617-1201, ISSN(p): 2617-1732
Website: https://www.sumerianz.com

Original Article Open Access

An Application of the "5E's" Instructional Treatment to Teaching the Concept of Fuzzy Set

Michael Gr. Voskoglou
Graduate Techmological Educational Institute of Western Greece, Greece

Abstract

The fuzzy set theory has been rabidly developed during the last years and has found many and important applications to everyday life, science, technology and equqation. As a result, elements of fuzzy mathematics have been already introduced in the curricula of studies of certain university departments.. In the article at hand the "5 E's" instructional model is applied for teaching the concept of fuzzy set to university students. This model is based on the principles of social constructivism for learning and has become very popular during the last decades, especially in school education, for teaching mathematics. The outcomes of the qualitative and quantitative analysis of its application for teaching the fuzzy set performed here demonstrate, apart from a very satisfactory student performance, better motivation and deeper understanding of the subject, as well as greater self-efficacy in tackling the corresponding problems with respect to previous cases, where the traditional teacher-centered method had been used.. Therefore, the application of the "5 E's" model for teaching various other fundamental mathematical concepts at universi8ty level seems to be an interesting and promising area for further empirical research.
Keywords: Fuzzy set; Social constructivism; "5 E's "Instructional treatment.

1. Introduction

Fuzzy set theory, introduced by Zadeh (1965), and fuzzy logic that is based on it, have been rapidly developed during the recent years and have found important applications to almost all sectors of the human activity Klir and Folger (1988), Chapter 6, Voskoglou (2017), Chapters 5-8, Voskoglou (2018) As a result elements of fuzzy mathematics have been already introduced in the curricula of studies of certain university departments, in subjects like Mathematics, Physics, Engineering, Management, Economics, etc.

Although the principles of constructivism and of the socio-cultural theories for learning, the combination of which is known as social constructivism (Driver *et al.*, 1994; Ernest, 1998), have become very popular during the last decades in school education for teaching mathematics (Voskoglou, 2019a), in the university departments of positive sciences the majority of the instructors still prefer the traditional way of the teacher-centered mathematics instruction. However, recent research results (Lahdenpera *et al.*, 2019; Voskoglou, 2019b) suggest that the the application of methods based on ideas of the social constructivism could offer many advantages to the teaching and learning of mathematics at university level.

Those findings, as well as my personal experience of the student significant difficulties in nunderstanding the concept of fuzzy set, were the reasons for performing the present research concerning the application of the "5 E's" instructional trearment for teaching the concept of fuzzy set. The rest of the article is organized as follows: In Section 2 the headlines of the "5 E's" instructional model are presented. In Section 3 the application of this model for teaching the concept of fuzzy set is developed in detail and in Section 4 the quantitative analysis of the results obtained follows. The article closes with the general conclusions presented in Section 5.

2. The "5 E's" Instructional Model

The application of the socio-constructive theories for learning in teaching mathematics have started during the 1980's, when the failure of the introduction of the "new mathematics" to the school curricula had already become more than evident to everybody.

The idea that knowledge is a human construction supported by the experience, first stated by Vico in the 18th century and further extended by Kant, affected greatly the epistemology of Piaget, who is considered to be the forerunner of the theory of constructivism for the process of learning. This theory, which appeared formally by von Glasersfeld (1987), is based on the following two principles:

i. Knowledge is not passively received from the environment, but it is actively constructed by synthesizing past knowledge and experience with the new information.

ii. The "coming to know" is a process of adaptation based on and constantly modified by the individual's experience of the world.

On the other hand, the socio-cultural theory for learning is based on the Vygotsky's ideas claiming that knowledge is a product of culture and social interaction. Learning takes place when the individuals engage socially to talk and act about shared problems or interests (Elbers, 2003; Goos, 2014; Jaworski, 2006; Sfard, 2000; Wenger, 1998).

The "5 E's" is an instructional model based on the principles of social constructivism that has become recently very popular for teaching mathematics (Enhancing Education, 2019). Each of the 5 E's describes a phase of learning which begins with the letter "E". Those phases are the following:

i. Engage: This is the starting phase which connects the past with the present learning experiences and focuses student thinking on the learning outcomes of the current activities.

ii. Explore: During this phase students explore their environment to create a common base of experiences by identifying and developing concepts, processes and skills.

iii. Explain: In this phase students explain and verbalize the concepts that they have been explored and they develop new skills and behaviors. The teacher has the opportunity to introduce formal terms, definitions and explanations for the new concepts and processes and to demonstrate new skills or behaviors.

iv. Elaborate: In this phase students develop a deeper and broader conceptual understanding and obtain more information about areas of interest by practicing on their new skills and behaviors.

v. Evaluate: This is the final step of the "5E's" instructional model, where learners are encouraged to assess their understanding and abilities and teachers evaluate student skills on the new knowledge.

The "5 E's" model allows students and teachers to experience common activities, to use and build on prior knowledge and experience and to continually assess their understanding of a concept. Although it has been mainly applied in school education (Keeley, 2017) the 5 E's can be used with students of all ages, including adults (Hee *et al.*, 2013).

3. Teaching the Concept of the Fuzzy Set

A typical way to teach the concept of fuzzy set to university students is to start with its formal definition in terms of the membership function, to proceed to a number of suitable examples, to generalize the ordinary operations of crisp sets to fuzzy sets, etc. However, using this method with my engineering students, I have realized that students faced significant difficulties in understanding the concept of fuzzy set, confusing in particular the membership degrees with the probabilities. This gave me the impulsion to design a method of teaching the fuzzy set based on the "5E's" instructional model, the headlines of which are the following:

Engage: The student attention is turned to the fact that definitions appear frequently in the everyday life having no clear boundaries; for example "the high mountains of a country", "the young people of a city", "the good players of a foot-ball club", etc. This phenomenon creates a kind of uncertainty due to imprecision. Probability theory, although it is suitable to deal with the uncertainty created by randomness, it is unable to tackle the situations of imprecision. Therefore, there is a need for a mathematical formulation of such kind of situations, in order to be able to solve the corresponding problems.

Explore: The characteristic property of a crisp set is aways defined in an explicit way. For example, assuming that a mountain is considered to be high, if its height is greater than 2000 meters, it is straightforward to form the set of "the high mountains of a country" by selecting them from the universal set of all its mountains. The question is what we can do if the definition of the characteristic property of the given set has no clear boundaries, i.e. if we have to deal with a fuzzy set.. The instructor leaves students to express their own ideas about it, and starting from the characteristic function of a crisp set, which takes only the values 0 and 1, "leads" properly the discussion to the notion of the membership function of a fuzzy set that takes values from the universal set to the real interval [0, 1]. For example, in this way ALL the mountains of a country belong to the fuzzy set of its high mountains with membership degrees tending to 1 if their height is considerably great or tending to 0 if their height is low. Therefore a mountain is not characterized only as high or not high according to the law of the excluded middle of the classical logic, but with various other linguistic characterizations (labels) like almost high, rather high, rather not high, probably not high, etc., each one corresponding to a numerical value of the imterval [0, 1]. This treatment gives to students a first empirical idea of the concept of fuzzy set and of the infinite-valued fuzzy logic generated by it.

Explain: At this phase the instructor has the opportunity to introduce the formal definition of a fuzzy set in the universal set U, as the set of all ordered pairs $\{(x, m(x)): x \in U\}$, where $m: U \rightarrow [0,1]$ is the corresponding membership function. For reasons of simplicity one could identify a fuzzy set with its membership function.

Attention is given to the fact that the definition of the membership function is not unique depending on the observer's personal goals. However, it must be always compatible to the common logic, otherwise the fuzzy set does not give a reliable representation of the corresponding real situation. For example, in the fuzzy set of the high mountains it is not reasonable to define the membership degree of a mountain with height 500 meters to be near to 1, neither it is reasonable to define the membership degree of a mountain of height 3000 meters to be near to 0.

Suitable examples of fuzzy sets follow with discrete and continuous or piece – wise continuous functions (triangulat, trapezoidal, etc.) and the students are asked to choose the most suitable membership function for certain fuzzy sets.among a number of given functions.

A crucial thing to be clarified here is that, although probabilities and membership degrees are functioning in the same interval [0, 1], they differ significantly to each other. For example, the expressions "The probability for Mary to be tall is 80%" and "The membership degree of Mary in the fuzzy set of the tall people is 0.8" have a completely diffent meaning. The former means that Mary, being an unknown to the observer person, is either tall or not tall (according to the law of the excluded middle), but her given outline suggests that she is very probably tall. On the contrary, the latter means that Mary could be characterized as a rather tall person.

A suitable example is also needed to clarify that, whereas the sum of the probabilities of the single events (singlenton sets of the sample space) is always equal to 1, this is not necessarily true in a fuzzy set for the sum of the membership degrees of the elements of the universal set. Therefore, a probability distribution could be used to define a membership function, but the converse does not hold in all cases.

Elaborate: The student opinion is asked here whether or not fuzzy logic is compatible to the classical bi-valued logic of Aristotle. The first remark on it is that obviously a crisp set can be considered as a fuzzy set with membership function its characteristic function. Next, given two fuzzy sets A and B of U with membership functions m_A and m_B respectively, the instructor defines A to be a subset of B, if $m_A(x) \leq m_B(x)$, for all x in U. Then it is easy for the students to check that this definition generalizes the ordinary definition of crisp subsets. Further, a new fuzzy set is defined by the instructor with membesship function $m\{x\} = \min \{m_A(x), m_B(x)\}$ and the students are asked to recognize that this definition generalizes the ordinary definition of the intersection of two crisp sets. In the same way the union of two fuzzy sets is defined to be the fuzzy set with membership function $m\{x\} = \max \{ m_A(x), m_B(x)\}$, the compement of the fuzzy set A is the fuzzy set with membership function $1 - m_A(x)$, etc. Therefore, we have a first strong indication that fuzzy logic is actually a generalization and a complement of the classical logic.

The instructor emphasizes to students that almost all logical operations of the traditional logic can be extended and new ones are created in fuzzy logic, which gives to the scientists the necessary tools to model under imprecise conditions and therefore to tackle mathematically problems stated in our natural language. He also reviews in a simple way some of the most important practical applications that fuzzy logic has already found in real life, science and technology problems (fuzzy decision making, fuzzy control and programming, robotics, etc.).

Evaluate: At the end of the teaching process a number of exercises and problems analogous to those solved in the classroom was given to students on the purpose of checking at home their understanding of the subject. In the next lecture students solved them on the board and discussed with the instructor their difficulties and doubts. They were also asked to reproduce orally the already taught definitions, concepts and processes.

The qualitative analysis of the student behaviors gave a very satisfactory impression, much better than that obtained in previous cases, where the traditional teaching process had been followed. Apart from their good general performance, students demonstrated a deeper understanding of the subject, a better motivation and a greater self-efficacy in tackling the corresponding problems.

4. Quantitative Analysis

A week later a written test was performed in the classroom involving some theoretical questions, exercises and problems. The student performance in the test was assessed with the linguistic grades: A = Excellent, B = Very Good, C = Good, D = Satisfactory and F = Failed. The linguistic assessment, which is often used in practice, prevents dilemmas about the exact numerical score corresponding to each student's performance This reduces significantly the student anxiety caused by the very existence of the numerical scores that has a permanent effect on the nature and direction of student learning. However, the linguistic assessment does not allow the application of the conventional statistical methods for analyzing the student performance and in particular the calculation of the mean value of all student scores for evaluating their mean performance. In earlier works we have developed two equivalent methods for estimating the student mean performance by using as tools triangular fuzzy numbers and grey numbers respectively Voskoglou (2019c), Sections 5.2 and 6.2). Here, on the purpose of avoiding to make the present work too technical, we shall use instead the Grade Point Average (GPA) index for evaluating the student overall performance.

Let n the total number of students and let n_A, n_B, n_C, n_D and n_F be the numbers of students whose performance was characterized as excellent, very good, good, satisfactory and as a failure respectively.Then the GPA index is calculated by the formula Voskoglou (2017), Chapter 6, p. 125):

$$GPA = \frac{0n_F + 1n_D + 2n_C + 3n_B + 4n_A}{n} \quad (1)$$

In other words GPA is a weighted average assessing the student quality performance by assigning greater coefficients (weights) to the higher grades. Obviously GPA takes values between 0 (worst performance, $n_F = n$) and 4 (best performance, $n_A = n$).

The outcomes of the written test are depicted in Table 1

Table-1. The outcomes of test

Grade	No. of students
A	15
B	29
C	13
D	14
F	4
Total	75

Therefore formula (1) gives that GPA = 2.56, which demonstrates a more than satisfactory overall student performance. Notice that in two previous cases, where the traditional teaching process had been followed, the corresponding values of the GPA index have found to be less than 2 (1.95 and 1.78 respectively) and in one case less than 1.5 (1.43).

5. Discussion and Conclusions

Learning is an integral part of our everyday lives. The problem is not that we do not know this, but rather that we do not have very systematic ways of talking about this familiar experience. Mathematics teaching is intended to promote the learning of mathematics. But, while theory provides us with lenses for analyzing learning, the position of mathematics teaching remains theoretically anomalous and underdeveloped. We might see one of the problems to lie in the relationships between learning, teaching and the practice of teaching. Theories help us to analyze, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for practice.

In the present article we have applied the "5 E's" model for teaching the concept of fuzzy set to engineering students. This is an instructional treatment based on the principles of social constructivim for learning that have become very popular in mathematics education during the last decades. From what it has been discussed here, it becomes evident that the application of the "5 E's" for teaching mathematics is a time-consuming method. Therefore, it cannot be used in practice as a general teaching method in cases where an extendend amount of mathematical material has to be covered in a certain time, as it usually happens at university level. However, its use in special cases, where funamental new concepts are introduced, it could be beneficial to students for a better motivadion and a deeper understanging of the subject. The case of fuzzy sets, presented in this work, gives a strong support to the above argument. Consequently, the application of the "5 E's" method at university level for introducing various other fundamental mathematical concepts, like the limit, the derivative, the integral, etc. is a promising and interesting area for further empirical research.

References

Driver, R., Asoko, H., Leach, J., Mortimer, E. and Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher,* 23(7): 5-12.

Elbers, E. (2003). Learning and teaching mathematics in a community of inquiry. *Educational Studies inMathematics,* 54(1): 77-99.

Enhancing Education (2019). The 5 E's. Available: http://enhancinged.wgbh.org/research/eeeee.html

Ernest, P. (1998). *Social constructivism as a Philosophy of Mathematics.* State University of New York Press: Albany, New York.

Goos, M. (2014). *Communities of practice in mathematics teacher education, in s. Lerman (ed.), Encyclopedia of Mathematics Education.* Springer: Dordrecht, Netherlands. 82-84.

Hee, J. W., Lee, E. J., Park, H. J., Chang, A. K. and Kim, M. J. (2013). Use of the 5E learning cycle model combined with problem-based learning for fundamentals of a nursing course. *Journal of Nursing Education,* 52(12): 681-89.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education,* 9(2): 187-211.

Keeley, P. (2017). Embedding formative assessment into the 5e instructional model. *Science and Children,* 55(4): 28-31.

Klir, G. J. and Folger, T. A. (1988). *Fuzzy Sets, Uncertainty and Information.* Prentice-Hall: London, UK.

Lahdenpera, J., Postareff, L. and Ramo, J. (2019). Supporting quality of learning in university mathematics: A comparison of two instructional designs. *International Journal of Research in Undergraduate Mathematics Education,* 5(3): 75-96.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking an Learning,* 2(3): 157-89.

von Glasersfeld, E. (1987). *Learning as a constructive activity. In c. Janvier (ed), problems of representation in the teaching and learning of mathematics.* Lawrence Erlbaum: Hillsdale, N. J., USA. 3-17.

Voskoglou, M. G. (2017). *Finite markov chain and fuzzy logic assessment models: Emerging research and opportunities.* Createspace.com–Amazon: Columbia, SC, USA.

Voskoglou, M. G. (2018). Fuzzy logic: History, methodology and applications to education. *Sumerianz Journal of Education, Linguistics and Literature,* 1(1): 10-18.

Voskoglou, M. G. (2019a). Teaching and learning mathematics: Research and practice for the 21st century. *Sumerianz Journal of Education, Linguistics and Literature,* 2(4): 19-24.

Voskoglou, M. G. (2019b). Comparing teaching methods of mathematics at university level. *Education Sciences,* 9(3): 204.

Voskoglou, M. G. (2019c). Methods for assessing human-machine performance under fuzzy conditions. *Mathematics,* 7(3): 230.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press, UK: Cambridge.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control,* 8(3): 338–53.