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Application of Grey Numbers to Assessment of Case-Based Reasoning Systems

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Abstract

Case-Based Reasoning (CBR) is the process of solving new problems (usually with the help of computers) by adapting the solutions of similar (analogous) problems solved in the past. The CBR approach has got a lot of attention over the last 30-40 years, because as an intelligent-systems method enables information managers to increase efficiency and reduce cost by substantially automating processes. In the present work a method is developed using Grey Numbers (GNs) as a tool for accessing the effectiveness of CBR systems and examples are provided illustrating it. This new assessment method is proved to be equivalent with an analogous method using Triangular Fuzzy Numbers that has been developed in earlier author's works, but it has the advantage of reducing significantly the required computational burden. GNs, which are defined with the help of the closed intervals of real numbers, are indeterminate numbers whose range is known, but not their exact value, and they have found nowadays many and important applications in science, engineering and in the everyday life for handling approximate data.
Keywords: Problem-solving (PS); Analogical reasoning (AR); Case-based reasoning (CBR); Grey systems (GS); Grey numbers (GNs); Whitening, Fuzzy numbers (FNs); Triangular FNs (TFNs); Centre of gravity (CoG) defuzzification technique; Assessment methods.

1. Introduction

In Management a *system* is defined to be a set of interacting components forming an integrated whole and working together for achieving a common target. A factory, a hospital, a bank, etc. are common examples of systems, whereas in general one can distinguish between physical, biological, social, economic, engineering, abstract knowledge systems, etc. As a multi- perspective domain systems' theory serves as a bridge for an interdisciplinary dialogue between autonomous areas of study [1]. In the present work we deal with *Case-Based Reasoning (CBR) Systems*.

The assessment of a system's performance is a very important part of the systems' theory, because it enables the system's designer to correct its weaknesses and therefore to increase its effectiveness. When the performance of a system's components is evaluated with numerical scores, then the traditional way for assessing the system's *mean performance* is the calculation of the mean value of those scores. However, in order to comfort the user's existing uncertainty about the exact value of the numerical scores corresponding to each of the system's components, frequently in practice the assessment is made not by numerical scores but by qualitative linguistic expressions, like excellent, very good, good, etc, which makes the calculation of their mean value impossible.

A popular in such cases method for evaluating the overall system's performance is the calculation of the *Grade Point Average (GPA) index* [2]. However, GPA is a weighted average in which greater coefficients (weights) are assigned to the higher grades, which means that it reflects not the mean, as we wish, but the *quality* system's *performance*.

In order to overcome this difficulty we have utilized in earlier works the system's *total uncertainty* under fuzzy conditions (because of the qualitative assessment of its components) as a measure of its effectiveness [2]. This manipulation is based on a fundamental principle of the Information Theory according to which the reduction of a system's uncertainty is connected to the increase of information obtained by a system's activity. In other words, lower uncertainty indicates a greater amount of information and therefore a better system's performance with respect to the corresponding activity. However, this method needs laborious calculations, cannot give a precise qualitative characterization of a system's performance and, most importantly, it is applicable for comparing the performance of two different systems with respect to a common activity only under the assumption that the existing uncertainty is the same in the two systems before the activity. This was the reason for turning in later works to the use of *Fuzzy Numbers (FNs)* for the assessment of a system's mean performance under fuzzy conditions [2].

In the present work *Grey Numbers (GNs)* are used for assessing the performance of CBR systems. This method is proved to be equivalent to the use of a special form of FNs, the *Triangular FNs (TFNs)*, but it has the advantage of reducing significantly the required computational burden. Moreover, the GNs are defined easily with the help of the closed intervals of real numbers, in contrast to the TFNs that need the knowledge of some basic elements of the theory of *Fuzzy Sets (FS)*.

The rest of the paper is formulated as follows: Section 2 is devoted to a brief description of the CBR process. In Section 3 the assessment method with the TFNs is briefly recalled and the necessary information about GNs is given, needed for the understanding of the article. The new assessment method using GNs is developed in Section 4 and its equivalence to the method with TFNs is also proved. Examples with CBR systems illustrating those methods are presented in Section 5. The article closes with the conclusions of the present research and a short discussion on the perspectives of future research on the subject presented in Section 6.

2. Case-Based Reasoning

CBR is a recent theory for *Problem-Solving (PS)* and *Learning* in computers and people. Broadly construed it is the process of solving new problems based on the solutions of similar past problems, i.e. a kind of analogy making. A lawyer, who advocates a particular outcome in a trial based on legal precedents, a physician who treats a patient based on the treatment of previous patients suffering from the same disease, or an auto mechanic who fixes a car's engine by recalling another car that exhibited similar symptoms, are using CBR

The importance of *Analogical Reasoning* (*AR*), in human thinking has been recognized many years ago. In fact, there are many studies developed and many experiments performed on individuals by mathematicians, psychologists and other scientists about the AR process [3]. However, it is the CBR approach that has got a lot of attention over the last 30-40 years, because as an intelligent systems' method enables information managers to increase efficiency and reduce cost of many human activities by substantially automating processes, such as diagnosis, scheduling and design [3]. Notice that the term AR has been sometimes used as a synonymous to the typical CBR approach [4]. Nevertheless, it has also been frequently used to characterize methods that solve new problems based on past cases of *different domains* [5, 6], whereas the typical CBR methods focus on single-domain cases (a form of intra-domain analogy).

CBR is often used where experts find it hard to articulate their thought processes when solving problems. This happens because knowledge acquisition for a classical knowledge-based system would be extremely difficult in such cases, and is likely to produce incomplete or inaccurate results. When using CBR the need for knowledge acquisition can be limited to establishing how to characterize *cases*, i.e. the analogous problems' situations. A *case-library* can be a powerful corporate resource allowing everyone in an organization to tap in it, when handling a new problem. A CBR *system,* usually designed and functioning with the help of computers, allows the case-library to be developed incrementally, while its maintenance is relatively easy and can be carried out by domain experts.

In CBR the term PS is used in a wide sense, which means that it is not necessarily the finding of a concrete solution to an application problem, it may be any problem put forth by the user. For example to justify or criticize a proposed solution, to interpret a problem's situation, to generate a set of possible solutions, or generate explanations in observable data, are also PS situations. Many experts distinguish between two styles of CBR, the *PS style* and the *interpretive style*. The PS style can support a variety of tasks including planning, diagnosis, help-desk applications, assessment, design, etc.; e.g. in Medicine [7], Industry [8], Robotics [9], etc. The interpretive style is useful for classification, evaluation or justification of a solution, argumentation and for the projection of effects of a decision. Lawyers and managers making strategic decisions use the interpretive style [10]. Organizations as diverse as IBM, VISA International, Volkswagen, British Airways and NASA have already made use of CBR in fields like customer support, quality assurance, aircraft maintenance, process planning, and many more that are easily imaginable.

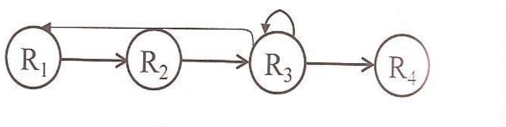
The coupling of CBR to learning occurs as a natural by-product of PS. When a problem is successfully solved, the experience is retained in order to solve similar problems in future. When an attempt to solve a problem fails, the reason for the failure is identified and remembered in order to avoid the same mistake in future. Thus CBR is a cyclic and integrated process of solving a problem, learning from this experience, solving a new problem, etc.

CBR has been formalized for purposes of computer and human reasoning as a four steps process, often referred as the "*four R's*". These steps involve:

- R_1: *Retrieve* the most similar to the new problem past case.
- R_2: *Reuse* the information and knowledge of the retrieved case for the solution of the new problem.
- R_3: *Revise* the proposed solution.
- R_4: *Retain* the part of this experience likely to be useful for future problem solving.

The first three of the above steps are not linear, characterized by a backward - forward flow among them. A simplified flow - chart of the CBR process, which is adequate for the purposes of the present paper, is presented in Figure 1 below:

Figure-1. A simplified flow-chart of the CBR process



A detailed functional diagram illustrating the four steps of the CBR process is presented in Voskoglou and Salem [3].

CBR traces its roots in Artificial Intelligence to the work of Roger Schank and his students at Yale University, U.S.A. in the early 1980's. Schank [11] model of *dynamic memory* was the basis of the earliest CBR systems that might be called case-based reasoners, the Kolodner [12] CYRUS and the Lebowitz [13] IPP. More details about the CBR methodology, history and applications can be found in [3, 14] and in the relevant references given in the previous papers.

As a general PS methodology intended to cover a wide range of real-world applications, CBR must face the challenge to deal with uncertain, incomplete and vague information. Correspondingly recent years have witnessed an increased interest in formalizing parts of the CBR methodology within frameworks of reasoning under uncertainty, and in building hybrid approaches by combining CBR with methods of uncertain and approximate reasoning; e.g. see [2] and its relevant references.

3. Mathematical Background

3.1 Triangular Fuzzy Numbers (TFNs)

It is assumed that the reader is familiar with the basic principles of the theory of *Fuzzy Sets (FS)* and the book of Klir and Folger [15] is proposed as a general reference on the subject. It is recalled that a FS on the set of the discourse U is a set A of ordered pairs $\{(x, m_A(x)), x \in U\}$, where $m_A : U \to [0, 1]$ is its *membership function*. The closer is the *membership degree* $m_A(x)$ of x in U to 1, the better x satisfies the characteristic property of A.

A *Fuzzy Number (FN)*, say A, is a FS on the set $\boldsymbol{R}$ of the real numbers, which is *normal* (i.e. there exists x in $\boldsymbol{R}$ such that $m_A(x) = 1$) and *convex* (i.e. all its *a-cuts* $A^a = \{x \in U: m_A(x) \geq a\}$, a in [0, 1], are closed real intervals) and whose membership function $y = m_A(x)$ is a *piecewise continuous function*. For general facts on FNs we refer to the book of Kaufmann and Gupta [16]

A *Triangular FN* (a, b, c), with a, b, c real numbers such that $a < b < c$ is the simplest form of a FN representing mathematically the fuzzy statement that “the value of b lies in the interval $[a, c]$”. The membership function $y = m(x)$ of (a, b, c) is zero outside the interval $[a, c]$, while its graph in $[a, c]$ consists of two straight line segments forming a triangle with the OX axis.

Therefore we have:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \ or \ x > c \end{cases}$$

Using elementary methods of Analytic Geometry it is straightforward to check [2] that the coordinates (X, Y) of the *Centre of Gravity (CoG)* of the graph of the TFN $A = (a, b, c)$ are calculated by the formulas

$$X(A) = \frac{a+b+c}{3}, \quad Y(A) = \frac{1}{3} \qquad (1)$$

The first of formulas (1) can be used to defuzzify the TFN A (*CoG defuzzification technique)*, e.g. see [17], i.e. to represent it by a crisp number.

There are known two equivalent general methods for defining *arithmetic operations* on FNs (Kaufmann & Gupta, 1991). Those methods lead to the following simple rules for the *addition* and *subtraction* of TFNs:

Let $A = (a, b, c)$ and $B = (a_1, b_1, c_1)$ be two TFNs. Then one defines:

- The *sum* $A + B = (a+a_1, b+b_1, c+c_1)$.
- The *difference* $A - B = A + (-B) = (a-c_1, b-b_1, c-a_1)$, where $-B = (-c_1, -b_1, -a_1)$ is defined to be the *opposite* of B.

On the contrary, the *product* and the *quotient* of A and B are FNs, which are not TFNs in general, apart from some special cases.

The following two *scalar operations* can be also defined:

- $k + A = (k+a, k+b, k+c)$, $k \in \boldsymbol{R}$
- $kA = (ka, kb, kc)$, if $k>0$ and $kA = (kc, kb, ka)$, if $k<0$, $k \in \boldsymbol{R}$.

3.2 The Assessment Method using TFNs

For the better understanding of the present work our assessment method using TFNs developed in earlier works [2] is recalled here in brief.

For this, let A_i, i = 1, 2,…, n be given TFNs, where n is a non negative integer, $n \geq 2$. Then, we define the *mean value* of the A_i’s to be the TFN $A = \frac{1}{n}(A_1 + A_2 + \dots + A_n)$.

The qualitative grades A = excellent, B = very good, C = good, D = fair and F = unsatisfactory are considered for the assessment of a system’s performance and a scale of numerical scores from 1 – 100 is assigned to them as follows: A (85 - 100), B (75 - 84), C (60 – 74), D (50 - 59) and F (0 - 49) [1].

Then, each of the above grades can be represented by a TFN, denoted for simplicity by the same letter, as follows: A = (85, 92.5, 100), B = (75, 79.5, 84), C (60, 67, 74), D (50, 54.5, 59) and F (0, 24.5, 49). The middle entry of each of the above TFNs is equal to the average of its other two entries. In other words, if A (a_1, b_1, c_1), B (a_2, b_2, c_2),…., F(a_5, b_5, c_5), then $b_i = \frac{a_i + c_i}{2}$, i = 1, 2, 3, 4, 5.

In order to assess the total system’s effectiveness, the performance of each of the system’s components is evaluated by one of the above five qualitative grades, which means that one of the TFNs A, B, C, D, F can be assigned to each component.

Let n be the total number of the system’s components and let n_X be the number of the components corresponding to the TFN X, where X = A, B, C, D, F. Then the mean value M of all those TFNs is equal to the TFN

$$M(a, b, c) = \frac{1}{n}(n_A A + n_B B + n_C C + n_D D + n_F F) \qquad (2).$$

Since the calculation of the mean value of the qualitative grades is not possible, it looks logical to consider the TFN M as the fuzzy representative for evaluating the system's mean performance. Replacing the values of the TFNs A, B, C, D and F in equation (2) it is straightforward to check that

$$M(a,b,c)=\left(\frac{85n_A+75n_B+60n_C+50n_D+0n_F}{n}, \frac{92.5n_A+79.5n_B+67n_C+54.5n_D+24.5n_F}{n},\right.$$

$$\left.\frac{100n_A+84n_B+74n_C+59n_D+49n_F}{n}\right)$$. Then, equation (1) gives that

$$X\,(\mathrm{M}) = \frac{a+b+c}{3} = \frac{92.5n_A+79.5n_B+67n_C+54.5n_D+24.5n_F}{n} = \frac{a+c}{2} = b \quad (3).$$

The value of X (M) provides a crisp representation of the TFN M evaluating the system's mean performance.

3.3 Grey Numbers

Frequently in the everyday life, as well as in many applications of science and engineering, a system's data cannot be easily determined precisely and in practice estimates of them are used. The reason for this is that in large and complex systems, like the socio – economic, the biological ones, etc., many different and constantly changing factors are usually involved, the relationships among which are indeterminate, making their operation mechanisms to be not clear.

Nowadays two are the main tools for handling such approximate data: *Fuzzy Logic (FL)*, which is based on the notion of FS initiated by Zadeh [18] and the theory of *Grey System (GS)* initiated by Deng [19]. The GS theory was mainly developed in China and it has found nowadays important applications in agriculture, economy, management, industry, ecology, environment, meteorology, geography, geology, earthquakes, history, military affairs, sports, traffic, material science, biological protection and in many other fields of the human activity; see Deng [20] and its relevant references.

Roughly speaking, a GS is understood to be any investigated system with "poor" information**.** More explicitly, the systems which lack information, such as structure message, operation mechanism and behaviour document, are referred to as GSs. For example, the human body, the world economy, etc., are GSs. Usually, on the grounds of existing grey relations and elements one can identify where "grey" means poor, incomplete, uncertain, etc.

The aim of the GS theory is to provide techniques, notions and ideas for analyzing latent and intricate systems, including the establishment of non-function models, the development of a grey process replacing an existing stochastic process, the transformation of disorderly raw data into a more regular series by grey generating techniques instead of modelling with the original data; grey decision making, grey forecasting control replacing classical control, the study of feeling and emotion functions and fields with whitening functions, the study in general of grey mathematics instead of classical mathematics, etc [20].

An effective tool for handling the approximate data of a GS is the use of GNs. A GN is an indeterminate number whose probable range is known, but which has unknown position within its boundaries. The GNs are defined with the help of the closed real intervals. More explicitly, if $\boldsymbol{R}$ denotes the set of real numbers, a GN, say A, can be expressed mathematically by

$$A \in [a, b] = \{x \in \boldsymbol{R} : a \leq x \leq b\}.$$

If $a = b$, then A is called a *white number* and if $A \in (-\infty, +\infty)$, then it is called a *black number***.** The GN A may enrich its uncertainty representation with respect to the interval [a, b] by a *whitening function* f: [a, b] $\rightarrow$ [0, 1] defining a *degree of greyness* f(x) for each x in [a, b]. The closer is f(x) to 1, the greater the probability for x to be the representative real value of the GN A. For general facts on GNs we refer to Liu and Lin [21].

The well known arithmetic of the real intervals [22] has been used to define the basic arithmetic operations among the GNs. More explicitly, if $A \in [a_1, a_2]$ and $B \in [b_1, b_2]$ are given GNs and k is a real number, one defines:

- *Addition* by $A + B \in [a_1 + b_1, a_2 + b_2]$
- *Subtraction* by $A - B = A + (-B) \in [a_1 - b_2, a_2 - b_1]$, where $-B \in [-b_2, -b_1]$ is defined to be the *opposite* of B.
- *Multiplication* by $A \times B \in [min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}, max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}]$
- *Division* by $A : B = A \times B^{-1} \in [min\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\}, max\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\}]$, where $0 \notin [b_1, b_2]$ and $B^{-1} \in [\frac{1}{b_2}, \frac{1}{b_1}]$ is defined to be the *inverse* of B .
- *Scalar multiplication* by $kA \in [ka_1, ka_2]$, if $k \geq 0$ and by $kA \in [ka_2, ka_1]$, if $k < 0$.

Observe that $B + (-B) \in [b_1 - b_2, b_2 - b_1] \neq [0, 0] = 0$, $B + (-B) \neq (-B) + B \neq 0$ and

$B \times B^{-1} = B^{-1} \times B \in [\frac{b_1}{b_2}, \frac{b_2}{b_1}] \neq [1, 1] = 1$.

The white number with the greatest probability to be the representative real value of the GN $A \in [a, b]$ is denoted by W(A). The technique of determining the value of w (A) is called *whitening* of A. One usually defines

W(A) = $(1- t)a + tb$, with t in [0, 1]. This is known as the *equal weight whitening*. In this case, if the distribution of A is unknown (i.e. no whitening function has been defined for A), one takes t = $\frac{1}{2}$, which gives that W(A) = $\frac{a+b}{2}$ (4).

4. The Assessment Method with GNs

According to equation (3) at the end of Section 3.2 in order to calculate X(M) it is enough in practice to calculate the middle entry b only of the TFN M(a, b, c). This gives the impulse to search for a possible "formal" assessment procedure, similar to that of using TFNs, reducing the required computational burden, which led us to the utilization of GNs instead of TFNs for a system's assessment.

For this, considering again the numerical scores A (100-85), B(84-75), C (74-60), D(59-50), F(49-0) attached to the corresponding qualitative grades, we correspond to each grade a GN, denoted for simplicity with the same letter, as follows: A$\in$ [85, 100], B$\in$[75, 84], C$\in$ [60, 74], D$\in$[50, 59] and F$\in$[0, 49].

Next, assigning to each of the system's components the GN assessing its individual performance and keeping the same notation as in the case of the TFNs, we consider the *mean value*

$$M^* = \frac{1}{n}\,[n_A A + n_B B + n_C C + n_D D + n_F F] \quad (5)$$

of all those GNs as the representative of the system's mean performance.

But n_AA$\in$[85n_A, 100n_A], n_BB$\in$[75n_B, 84n_B], n_CC$\in$[60n_C, 74n_C], n_DD$\in$[50n_D, 59n_D] and n_FF$\in$[0n_F, 49n_F], therefore it turns out that M$\in$ [m_1, m_2], with

$$m_1 = \frac{85n_A + 75n_B + 60n_C + 50n_D + 0n_F}{n} \quad \text{and} \quad m_2 = \frac{100n_A + 84n_B + 74n_C + 59n_D + 49n_F}{n}.$$

Since the distributions of the GNs A, B, C, D and F are unknown, the same happens with the distribution of M*. Therefore, one can take

$$W(M^*) = \frac{m_1 + m_2}{2} \quad (6).$$

From equations (3) and (6) it turns out that X(M) = W(M*), which means that the assessment methods of a system's mean performance using as tools the TFNs or the GNs A, B, C, D and F respectively are *equivalent to each other*, providing the same assessment outcomes.

Moreover, one observes that in the extreme case where the maximal possible numerical score corresponds to each component for each grade, i.e. the n_A scores corresponding to A are 100, the n_B scores corresponding to B are 84, etc., the mean value of all those scores is equal to c or m_2 respectively. Also, in the opposite extreme case, where the minimal possible numerical score corresponds to each component for each linguistic grade, i.e. the n_A scores corresponding to A are 85, the n_B scores corresponding to B are 75, etc,. the mean value of all those scores is equal to a or m_1 respectively. Consequently, the assessment methods with the TFNs and the GNs give a reliable approximation of the system's mean performance and therefore they are useful when no numerical scores are used, but the system's performance is assessed by qualitative grades

5. Examples on the Assessment of CBR Systems

In this Section we provide two examples in which our models with GNs / TFNs presented in Sections 4 and 3.2 respectively are used for the assessment of the effectiveness of CBR systems. Our assessment approach is validated by comparing its outcomes in our applications with the corresponding outcomes of two classical assessment methods of the bi-valued logic, the calculation of the *mean values* and of the *GPA index*.

Example 1: Consider two CBR systems designed for help desk applications with their libraries containing 105 and 90 past cases respectively. The two systems' designers have supplied them with the same mechanism (software) that enables the assessment of the degree of success of each one of their past cases at each step of the CBR process, when used for the solution of new similar problems. Table 1 depicts the degree of success of their past cases in each of the three first steps of the CBR process

Table-1. Assessment of the past cases of the CBR systems FIRST SYSTEM

Steps	F	D	C	B	A
R_1	0	0	51	24	30
R_2	18	18	48	21	0
R_3	36	30	39	0	0

Second System

Steps	F	D	C	B	A
R_1	0	18	45	27	0
R_2	18	24	48	0	0
R_3	36	27	27	0	0

Here we shall compare the quality performance of the two systems by calculating the GPA index and their mean performance by applying our methods with the TFNs or the GNs.

i) GPA index: Denote by y_i , i = 1, 2, 3, 4, 5 the frequencies of the CBR system's cases whose performance is characterized by F, D, C, B and A respectively, then the GPA index is calculated by the formula

$$GPA = y_2 + 2y_3 + 3y_4 + 4y_5 \quad (7).$$

In case of the ideal performance ($y_5 = 1$) we have GPA = 4, while in case of the worst performance ($y_1 = 1$) we have GPA = 0; therefore $0 \leq GPA \leq 4$. Consequently, values of GPA greater than 2 could be considered as corresponding to a more than satisfactory system's performance. In our case, the data of Table 1 give the following frequencies:

Table-2. Frequencies of the past cases for the CBR systems FIRST SYSTEM

Steps	y_1	y_2	y_3	Y_4	Y_5
R_1	0	0	$\frac{51}{105}$	$\frac{24}{105}$	$\frac{30}{105}$
R_2	$\frac{18}{105}$	$\frac{18}{105}$	$\frac{48}{105}$	$\frac{21}{105}$	0
R_3	$\frac{36}{105}$	$\frac{30}{105}$	$\frac{39}{105}$	0	0

Second System

Steps	y_1	y_2	y_3	Y_4	Y_5
R_1	0	$\frac{18}{90}$	$\frac{45}{90}$	$\frac{27}{90}$	0
R_2	$\frac{18}{90}$	$\frac{24}{90}$	$\frac{48}{90}$	0	0
R_3	$\frac{36}{90}$	$\frac{27}{90}$	$\frac{27}{90}$	0	0

Replacing the values of frequencies from Table 2 in formula (2) one finds the following values for the GPA index:

$$\text{First System:} \quad R_1: \frac{294}{105} = 2.8,\ R_2: \frac{177}{105} \approx 1.69,\ R_3: \frac{108}{105} \approx 1.03.$$

$$\text{Second System:} \quad R_1: \frac{189}{90} = 2.1,\ R_2: \frac{168}{90} \approx 1.87,\ R_3: \frac{81}{90} = 0.9$$

The above values of the GPA index show that the first system demonstrated a better quality performance at steps R_1 and R_3 (Retrieve, Revise). Whereas the second one demonstrated a better performance at R_2 (Reuse). Further, the two systems' performance was proved to be more than satisfactory in R_1 and less than satisfactory in the other two steps, being worse at R_3. This was logically expected, since the success in each step depends on the success in the previous steps. Notice that the two systems' performance at the last step R_4 was not examined, since, as we have seen in Section 2, all past cases, even the unsuccessful ones, are retained in a system's library for possible use in future with related new problems; the unsuccessful ones to help for exploring possible reasons of failure to find a solution for the new problem.

Finally, the mean values of the GPA index for the two systems at the three steps R_1, R_2 and R_3 are approximately equal to 1.84 and 1.62 respectively, showing that the first system demonstrated a better overall quality performance.

ii) Use of the TFNs: From the data of Table 1 one finds that for the first system and in step R_1 we have 51 TFNs equal to C(60, 67, 74), 24 TFNs equal to B(75, 79.5, 85) and 30 TFNs equal to A(85, 92.5, 100). The mean value of all those TFNs, denoted for simplicity by the same letter R_1, is equal to

$R_1 = \frac{1}{105}(51C + 24B + 30A) = \frac{1}{105}[(3060, 3417, 3774) + (1800, 1908, 2016) + (2550, 2775. 3000) = \frac{1}{105}(7410, 8100, 8790) \approx (70.57, 77.14, 83.71)$.

Therefore, from equation (3) one gets that $X(R_1) = 77.14$, which shows that the first system demonstrated a very good (B) performance at step R_1.

In the same way one calculates for the first system the mean values

$$R_2 = \frac{1}{105}(18F + 18D + 48C + 21B) \approx (51, 60.07, 69.14) \text{ and}$$

$R_3 = \frac{1}{105}(36F + 30D + 39C) \approx (36.57, 48.86, 61.14)$, thus obtaining the analogous conclusions for the system's performance at the steps R_2 and R_3 of the CBR process.

Finally, the overall system's performance can be assessed by the mean value

$$R = \frac{1}{3}(R_1 + R_2 + R_3) \approx (52.71, 62.02, 71.33),$$

Therefore, since $X(R) = 62.02$, the system demonstrated a good (C) mean performance.

A similar argument gives for the second system the values $R_1 = (62.5, 68.25, 74)$,

R_2 ≈(45.33, 55.17, 65), R_3 = (33, .46.25, 59.5) and R ≈ (46.94, 56.56, 66.17), thus obtaining the analogous conclusions for its mean performance at each step of the CBR process and its overall mean performance.

iii) Use of the GNs: According to this approach in step R_1 we have 51 GNs equal to C ∈ [60,74], 24 GNs equal to B ∈ [75, 84] and 30 GNs equal to A ∈ [85, 100]. The mean value of all those GNs, denoted by R_1*, is equal to

$$R_1^* = \frac{1}{105}(51C + 24B + 30A) \in [70.57, 83.71].$$

$$\text{Therefore, } W(R_1^*) = \frac{70.57 + 83.71}{2} = 77.14, \text{ etc.}$$

As we have seen in Section 4 this approach provides in general the same assessment outcomes with the use of TFNs, but, as it becomes more evident from the present example, it reduces significantly the required computational burden. Table 3 depicts the assessment outcomes obtained in this example

Table-3. Outputs of the Assessment Methods Used in Example.1

METHOD	OUTPUT
GPA index	The first system demonstrated a better overall quality performance and a better quality performanceat steps R_1 and R_3, while the second system performed better at step R_2
TFNs /GNs	The first system demonstrated a better mean performance at all the steps of the CBR process

The outcomes of Table 3 give emphasis to the fact that the assessment of a system's quality performance could lead to different outcomes from the assessment of its mean performance (step R_2).

Example 2: Six different users of a CBR system ranked with scores from 0-100 the effectiveness of its following five past cases for solving new related problems:

C_1 (Case 1)**:** 43, 48, 49, 49, 50, 52, C_2**:** 81, 83. 85, 88, 91, 95, C_3**:** 76, 82, 89, 95, 95, 98, C_4: 86, 86, 87, 87, 87, 88, C_5: 35, 40, 44, 52, 59, 62.

The *mean value* of the given, 6X5 = 30 in total numerical scores is approximately equal to 72.07 demonstrating a good (C) mean performance of the system with respect to the above five past cases. For reasons of comparison of the assessment outcomes the system's mean performance will also be calculated here by using our model with the GNs.

In fact, the given numerical scores correspond to 14 GNs equal to the GN A, 4 equal to B, 1 equal to C, 4 equal to D and 7 GNs equal to the GN F. The mean value of the above GNs is equal to

$$M^* = \frac{1}{30}(14A + 4B + C + 4D + 7F) \in [60.33, 79.63].$$

Therefore, from equation (6) one obtains that *W*(M*) = 69.98. Consequently the CBR system demonstrates a good mean performance with respect to the given five past cases. However the exact score corresponding to the system's mean performance is equal to the mean value 72.02 of the given numerical scores. In concluding, the assessment model using GNs (or TFNs), although it gives a good approximation of the system's mean performance, it is actually useful only when the effectiveness of the system's past cases is evaluated by qualitative grades and not by numerical scores, because in this case the calculation of the mean value of those grades is not possible..

6. Conclusions and Discussion

A method using GNs as tools was developed in the present research for assessing a system's mean performance, which is useful when using qualitative grades and not numerical scores for this purpose. This new method was proved to be equivalent with an analogous method using TFNs instead of GNs developed in earlier works, but it reduces significantly the required computational burden, since it requires the calculation of two components only (instead of three in case of the TFNs) for obtaining the mean value of the GNs. Examples were also presented on the assessment of CBR systems illustrating our results and showing that the system's quality performance, calculated by the traditional GPA index, may lead to different assessment conclusions.

Although the enormous development of technology during the last years makes easier and more comfortable the human life, it creates in parallel more and more complicated artificial systems, which are difficult to be managed by the traditional scientific methods. As a result, the applications of FL and of the GS theory have been rapidly expanded nowadays covering almost all sectors of the human activities. In particular the FNs and the GNs have been proved to be effective tools in handling approximate and /or uncertain data, playing an important role in fuzzy mathematics and in GS theory respectively, analogous to the role played by the ordinary numbers in crisp mathematics. Therefore, the attempt to use FNs or / and GNs to other practical problems, apart from the assessment purposes, in science and technology and in the everyday life, looks as a very interesting and promising direction for future research on the subject. Note that such efforts have been already started by the present author on solving equations, systems of equations and linear programming problems with fuzzy or grey data, connected to real life applications [23].

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