



Mathematical Derives of Evolutionary Algorithms for Multiple Criteria Decision Making

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Abstract

In multiple criteria decision making (MCDM) with interval-valued belief distributions (IVBDs), individual IVBDs on multiple criteria are combined explicitly or implicitly to generate the expected utilities of alternatives which can be used to make decisions with the aid of decision rules. Optimization models are usually constructed to implement such combination. To analyze a MCDM problem with a large number of criteria and grades used to profile IVBDs, effective algorithms are required to find the solutions to the optimization models within a large feasible region. We anticipate experimental results will indicate that particle swarm optimization algorithm is the best one to combine individual IVBDs and generate the minimum and maximum expected utilities of alternatives among the four algorithms.

Keywords: Multiple criteria decision making; Interval-valued belief distribution; Evolutionary algorithm; Accuracy; Efficiency.

1. Introduction

In an era of Internet and Big Data, people's lifestyle has undergone an unprecedented revolution. People's life is filled with a lot of data, which makes people more informative than ever before and meanwhile makes people be faced with the dilemma of choosing between deriving useful information and knowledge from data for real practices and abandoning their attempt to employ data in real problems. To follow the era, people usually select to find effective information and knowledge from various types of data and use them in practical cases. Such a choice contributes to the improvement of people's capability to handle complex problems. The choice also results in more uncertain environment associated with real problems than before due to the randomness, unavailability, noise, sparsity, and variety of data. In the environment people may have difficulties in directly finding overall solutions to real problems. A feasible way to overcome the difficulties is that real problems are analyzed from multiple different perspectives and then the relevant analyses are combined to generate the overall solutions to the problems. This way is considered as multiple criteria decision making (MCDM) in the uncertain environment. To effectively model and analyze uncertain MCDM problems, many attempts have been made with the help of different uncertain expressions. Representative expressions include intuitionistic fuzzy set [1], hesitant fuzzy linguistic set [2], hesitant fuzzy set [3], probabilistic linguistic set [4], belief distribution [5], interval-valued fuzzy set [6], interval-valued hesitant fuzzy linguistic set [7], interval-valued intuitionistic fuzzy set [8], interval-valued hesitant fuzzy set [9], interval type-2 fuzzy set Wang and Chen [10] and interval-valued belief distribution [11]. In theory, MCDM methods with these expressions seem to be sufficient for analyzing all real problems. From real cases or numerical examples in these studies, few methods are found to aim to solve large-scale problems with a large number of alternatives and criteria. In addition to this, when interval-valued assessments are adopted, such as interval-valued hesitant fuzzy elements or interval-valued belief distributions, the search space for finding solutions will increase exponentially along with the increase in the number of values in interval-valued hesitant fuzzy elements or the number of grades in interval-valued belief distributions. To find acceptable or satisfactory solutions within limited time, evolutionary computation provides a feasible and effective way. When MCDM problems are regarded as multi-objective optimization (MOO) problems constructed on a common set of variables, many evolutionary MOO approaches have been developed to find the optimum trade-off among criteria which is the most consistent with the preference of a decision maker [12-17]. Three ways, namely priori, interactive, and posteriori ones, are usually applied to combine the preferences of a decision maker with the MOO process [15, 16]. If the preferences of a decision maker are not considered in the MOO process, the results of the MOO are difficult to be satisfactory to the decision maker.

In practice, individual assessments on different criteria may not be always constructed on a common set of variables. For example, a radiologist determines whether a nodule of an inpatient is malignant form the perspectives of contour, echogenicity, calcification, and vascularity. It cannot be said that the judgments on the nodule with the

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consideration of contour and those with the consideration of any other perspective are made by the radiologist through the same set of variables (or features). As another example, when the same discipline in different universities is compared, many criteria will be considered, such as research projects, publications, awards, patents, social services, and excellent alumni. Individual assessments on different criteria are generated from different data rather than common data. When encountering these situations, a decision maker takes into account the individual assessments on all criteria synthetically rather than improves the values of most objectives and balances them to generate solutions. MCDM problems with large search spaces in these situations can also be solved by using evolutionary algorithms. For example, Javanbarg, *et al.* [18] used particle swarm optimization (PSO) algorithm Kennedy and Eberhart [19] to solve MCDM problems modeled by fuzzy analytic hierarchy process, and Chen and Huang [20] used PSO algorithm to solve MCDM problems modeled by interval-valued intuitionistic fuzzy numbers. Existing studies show that less attention is paid to the application of evolutionary algorithms to MCDM with different sets of variables used on different criteria. This makes it questionable whether MCDM methods with different ways to characterize different types of uncertain nature [3], [7-11] can be applied in solving MCDM problems with large search spaces. There is a gap between the solution requirements of large-scale MCDM problems and the relevant studies on effective solution approaches. Although there are few studies on the combination of evolutionary algorithms and MCDM with different sets of variables (e.g., [18-20], some important issues deserve investigation. The issues include: (1) why PSO algorithm is selected to be applied in MCDM; (2) whether PSO algorithm can be applied in MCDM with different types of uncertain expressions other than interval-valued intuitionistic fuzzy numbers and fuzzy triangular numbers; and (3) which evolutionary algorithm is of better performance when applied in MCDM with a specific type of uncertain expression. In fact, different evolutionary algorithms can be applied to solve the same real problem. For example, when determining the near-optimal scheme for recharging batteries at a battery swapping station, Wu *et al.* [21] used three representative evolutionary algorithms including genetic algorithm (GA) [22, 23], differential evolution (DE) algorithm Storn and Price [24] and PSO algorithm to find the minimum running cost. Their experimental results show that GA and DE algorithm achieve higher accuracy and lower efficiency than PSO algorithm, and specifically PSO algorithm fails to obtain the acceptable objective. Inspired by this, much attention should be paid to a key issue, which is to compare the accuracy Wu, *et al.* [21], Pei, *et al.* [25] and efficiency [14], [26] performances of different representative evolutionary algorithms for solving large-scale MCDM problems with specific types of uncertain expressions and different sets of variables. In this paper, to address the key issue, we aim to compare the accuracy and efficiency performances of four representative evolutionary algorithms, which are GA, DE algorithm, PSO algorithm, and gravitational search algorithm (GSA) [27], for analyzing MCDM problems modeled by interval-valued belief distributions (see Section 2). As the combination of individual interval-valued belief distributions is an important and necessary sub-process of MCDM, the processes of implementing the combination by using the four algorithms are presented. To conduct a fair comparison among the four algorithms, their original versions instead of their extensions are adopted in the processes. With the aid of the processes, experiments with different numbers of criteria and grades used to profile interval-valued belief distributions are performed to compare the accuracy and efficiency performances of the four algorithms for combining interval-valued belief distributions and generating the expected utilities. The comparative analysis of experimental results helps select the appropriate evolutionary algorithm to find satisfactory solutions to MCDM problems with interval-valued belief distributions within limited or acceptable time. A sensitivity analysis of the accuracy and efficiency performances of the four algorithms is provided to highlight the conclusion drawn from the comparative analysis

2. Preliminaries

2.1. Modeling of MCDM problems by using belief distributions In the evidential reasoning (ER) approach [28-31], which is a type of multiple criteria utility function method, belief distribution is used to characterize the uncertain preference of a decision maker. As belief distribution is a special case of interval-valued belief distribution, how to model MCDM problems by using belief distribution is recalled first. Suppose that alternative a_l ($l = 1, \dots, M$) is evaluated on criterion e_i ($i = 1, \dots, L$) by using a set of grades $\Omega = \{H_1, H_2, \dots, H_N\}$, which is increasingly ordered from worst to best. When $B(e_i(a_l))$ ($i = 1, \dots, L, l = 1, \dots, M$) is given, a belief decision matrix $L \times M$ is obtained. Assume that criteria weights are represented by $w = (w_1, w_2, \dots, w_L)$ such that $0 \leq w_i \leq 1$. Through combining individual belief distributions $B(e_i(a_l))$ ($i = 1, \dots, L, l = 1, \dots, M$) by using criteria weights and the ER rule [32], the overall belief distribution is obtained as $B(a_l) = \{(H_n, \beta_n(a_l)), n = 1, \dots, N; (\Omega, \beta\Omega(a_l))\}$. Similar to individual belief distribution, $\beta\Omega(a_l)$ represents the degree of aggregated global ignorance. It is not easy to directly compare the aggregated belief distributions of different alternatives in most cases. To facilitate comparison, $B(a_l)$ ($l = 1, \dots, M$) is transformed by using the utilities of grades $u(H_n)$ ($n = 1, \dots, N$) to the minimum and maximum expected utilities. From $u^-(a_l)$ and $u^+(a_l)$, a decision rule, such as Hurwicz rule [33, 34], can be used to aid in generating solutions.

2.2 Combination of belief distributions The contents in the above section show that the ER rule [35] is the key to find solutions to MCDM problems modeled by belief distributions, which is simply presented as follows. Definition 1 [35]. Given individual assessments $B(e_i(a_l))$ ($i = 1, \dots, L$) and their weights w_i , the combination result of the first i assessments is defined as

$$\{(H_n, \beta_{n,b(i)}(a_i)), n = 1, \dots, N; (\Omega, \beta_{\Omega,b(i)}(a_i))\}, \tag{1}$$

where

$$\beta_{n,b(i)}(a_i) = \frac{\hat{\beta}_{n,b(i)}(a_i)}{\sum_{n=1}^N \hat{\beta}_{n,b(i)}(a_i) + \hat{\beta}_{\Omega,b(i)}(a_i)}, \tag{2}$$

$$\beta_{\Omega,b(i)}(a_i) = \frac{\hat{\beta}_{\Omega,b(i)}(a_i)}{\sum_{n=1}^N \hat{\beta}_{n,b(i)}(a_i) + \hat{\beta}_{\Omega,b(i)}(a_i)}, \tag{3}$$

$$\bar{\beta}_{n,b(i)}(a_i) = \frac{\hat{\beta}_{n,b(i)}(a_i)}{\sum_{n=1}^N \hat{\beta}_{n,b(i)}(a_i) + \hat{\beta}_{\Omega,b(i)}(a_i) + \hat{\beta}_{P(\Omega),b(i)}(a_i)}, \tag{4}$$

$$\bar{\beta}_{\Omega,b(i)}(a_i) = \frac{\hat{\beta}_{\Omega,b(i)}(a_i)}{\sum_{n=1}^N \hat{\beta}_{n,b(i)}(a_i) + \hat{\beta}_{\Omega,b(i)}(a_i) + \hat{\beta}_{P(\Omega),b(i)}(a_i)}, \tag{5}$$

$$\bar{\beta}_{P(\Omega),b(i)}(a_i) = \frac{\hat{\beta}_{P(\Omega),b(i)}(a_i)}{\sum_{n=1}^N \hat{\beta}_{n,b(i)}(a_i) + \hat{\beta}_{\Omega,b(i)}(a_i) + \hat{\beta}_{P(\Omega),b(i)}(a_i)}, \tag{6}$$

$$\begin{aligned} \hat{\beta}_{n,b(i)}(a_i) &= [(1-w_i) \cdot \bar{\beta}_{n,b(i-1)}(a_i) + \bar{\beta}_{P(\Omega),b(i-1)}(a_i) \cdot w_i \beta_{n,i}(a_i)] + \bar{\beta}_{n,b(i-1)}(a_i) \cdot w_i \beta_{n,i}(a_i) + \\ &+ \bar{\beta}_{n,b(i-1)}(a_i) \cdot w_i \beta_{\Omega,i}(a_i) + \bar{\beta}_{\Omega,b(i-1)}(a_i) \cdot w_i \beta_{n,i}(a_i), \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{\beta}_{\Omega,b(i)}(a_i) &= [(1-w_i) \cdot \bar{\beta}_{\Omega,b(i-1)}(a_i) + \bar{\beta}_{P(\Omega),b(i-1)}(a_i) \cdot w_i \beta_{\Omega,i}(a_i)] + \\ &\bar{\beta}_{\Omega,b(i-1)}(a_i) \cdot w_i \beta_{\Omega,i}(a_i) \end{aligned} \tag{8}$$

and

$$\hat{\beta}_{P(\Omega),b(i)}(a_i) = (1-w_i) \cdot \bar{\beta}_{P(\Omega),b(i-1)}(a_i). \tag{9}$$

In Definition 1, $P(\Omega)$ represents the power set of Ω , and it is satisfied that $0 \leq \beta_{n,b(i)}(a_i) \leq \beta_{\Omega,b(i)}(a_i)$.

2.2. Modeling of MCDM problems by using interval-valued belief distributions Due to lack of sufficient data and knowledge or the nature of decision problems under consideration, a decision maker can only provide interval-valued belief distributions (IVBDs) as the evaluations of alternatives in some situations. For example, when a radiologist provides the diagnostic category in thyroid imaging reporting and data system published by Horvath et al. [36] for the thyroid nodule of an inpatient, he only reports the interval-valued cancer risk rather than precise cancer risk of the inpatient.

In this situation, individual IVBDs are represented. If it is satisfied that , the IVBDs are called valid ones [37]. Or else, they are invalid and cannot be used to generate valid belief distributions. Valid IVBDs are said to be normalized [37, 38] only when it is satisfied that

$$(\sum_{n=1}^N \beta_{n,i}^+(a_i) + \beta_{\Omega,i}^+(a_i)) - (\beta_{n,i}^+(a_i) - \beta_{n,i}^-(a_i)) \geq 1, n = 1, \dots, N, \tag{10}$$

$$(\sum_{n=1}^N \beta_{n,i}^+(a_i) + \beta_{\Omega,i}^+(a_i)) - (\beta_{\Omega,i}^+(a_i) - \beta_{\Omega,i}^-(a_i)) \geq 1, \tag{11}$$

$$(\sum_{n=1}^N \beta_{n,i}^-(a_i) + \beta_{\Omega,i}^-(a_i)) + (\beta_{n,i}^+(a_i) - \beta_{n,i}^-(a_i)) \leq 1, n = 1, \dots, N, \text{ and} \tag{12}$$

$$(\sum_{n=1}^N \beta_{n,i}^-(a_i) + \beta_{\Omega,i}^-(a_i)) + (\beta_{\Omega,i}^+(a_i) - \beta_{\Omega,i}^-(a_i)) \leq 1. \tag{13}$$

Normalized IVBDs are valid but valid IVBDs may be unnormalized [37]. From normalized individual IVBDs, a pair of optimization problems is constructed by using the ER rule to generate the aggregated IVBD $B(a)$.

$$\text{MIN/MAX } \beta_n(a_i) \tag{14}$$

$$\text{s.t. } \beta_{n,i}^-(a_i) \leq \beta_{n,i}^*(a_i) \leq \beta_{n,i}^+(a_i), \tag{15}$$

$$\beta_{\Omega,i}^-(a_i) \leq \beta_{\Omega,i}^*(a_i) \leq \beta_{\Omega,i}^+(a_i), \tag{16}$$

$$\sum_{n=1}^N \beta_{n,i}^*(a_i) + \beta_{\Omega,i}^*(a_i) = 1. \tag{17}$$

In the pair of optimization problems, $\beta_n(a_i)$ and $\beta_{\Omega}(a_i)$ represent decision variables, which form belief distributions limited to IVBDs. When the objective of the pair of optimization problems is changed to $\beta_{\Omega}(a)$ can be obtained. From the aggregated IVBD and the utilities of grades $u(H_n)$ ($n = 1, \dots, N$), the following optimization model is constructed to determine the minimum and maximum expected utilities $u^-(a)$ and $u^+(a)$.

$$\text{MIN } \sum_{n=1}^N \beta_n^*(a_i) u(H_n) + (\beta_1^*(a_i) + \beta_{\Omega}^*(a_i)) u(H_1) \tag{18}$$

$$\text{s.t. } \beta_n^-(a_i) \leq \beta_n^*(a_i) \leq \beta_n^+(a_i), \tag{19}$$

$$\beta_{\Omega}^-(a_i) \leq \beta_{\Omega}^*(a_i) \leq \beta_{\Omega}^+(a_i), \tag{20}$$

$$\sum_{n=1}^N \beta_n^*(a_i) + \beta_{\Omega}^*(a_i) = 1. \tag{21}$$

Solving this model, $\beta_n^*(a_i)$ represent decision variables, generates the optimal $u^-(a)$. When the objective of this model is changed to the optimal $u^+(a)$ can be obtained. If the aggregated IVBD is not required to analyze the decision problem under consideration, the optimization model shown in Eqs. (18)-(21) can be modified as the following one to determine $u^-(a)$ and $u^+(a)$.

$$\text{MIN } \sum_{n=1}^N \beta_n(a_i) u(H_n) + (\beta_1(a_i) + \beta_{\Omega}(a_i)) u(H_1) \tag{22}$$

$$\text{s.t. } \beta_{n,i}^-(a_i) \leq \beta_{n,i}^*(a_i) \leq \beta_{n,i}^+(a_i), \tag{23}$$

$$\beta_{\Omega,i}^-(a_i) \leq \beta_{\Omega,i}^*(a_i) \leq \beta_{\Omega,i}^+(a_i), \tag{24}$$

$$\sum_{n=1}^N \beta_{n,i}^*(a_i) + \beta_{\Omega,i}^*(a_i) = 1. \tag{25}$$

3. Four Evolutionary Algorithms for MCDM with IVBDs

The combination of individual belief distributions by using the ER rule to generate the aggregated belief distribution $B(a) = \{(H_n, \beta_n(a)), n = 1, \dots, N; (\Omega, \beta_{\Omega}(a))\}$ is implicitly involved in this optimization model. Similarly, the optimal $u^-(a)$ can be obtained from solving this model and the optimal $u^+(a)$ from solving the model with the objective.

When the number of criteria L and the number of grades N are large, solving the optimization problems shown in Eqs. (14)-(17) and (22)-(25) will become difficult. Evolutionary algorithms are helpful to find solutions to the optimization problems with large L and N . A key issue is to find the evolutionary algorithm with higher accuracy and efficiency performances among feasible algorithms. To address this issue, four evolutionary algorithms, namely GA, DE algorithm, PSO algorithm, and GSA, are adopted to make a comparison. The reason why the four evolutionary algorithms are selected is that many extensions of them are developed to handle real problems in different fields. For GA, its chromosome coding [39, 40] and structure [41] are improved, and it is used to conduct combinational dispatching decision [42], ischemic beat classification [43], and the generation of trading strategies for stock markets [44]. As to DE algorithm, its neighborhood-based mutation operator [45], dynamic parameter selection [46], self-adapting control parameters [47], and hybrid cross-generation mutation operation [48] are developed, and it is used to solve permutation flow shop scheduling problem [49] and periodic railway timetable scheduling problem [50]. With respect to PSO algorithm, its stability [51] and impacts of coefficients on movement patterns [52] are analyzed, and it is used to conduct cancer classification [53] and population classification in fire evacuation [54], and model the gene regulatory networks [55]. Concerning about GSA, its nearest neighbor scheme [56] is developed, and it is used to conduct feature selection for face recognition [57], unit commitment in power system operation [58], and parameter identification of water turbine regulation system [59]. In

3.1. Process of GA for Combining Individual IVBDs the GA Process for Combining Individual IVBDs is Presented as Follows

Step 1: Initialization.

For the pair of optimization problems in Eqs. (14)-(17).

Randomly generating NG chromosomes completes the initialization of the GA process. Assume that the maximum number of iterations is N_t . The crossover probability threshold C_GA and the mutation probability threshold M_GA are set as 0.6 and 0.1, respectively.

Step 2: Performance evaluation.

When the lower bound of $\beta_n(\alpha)$ is optimized, the fitness value of the chromosome, it is set as $f_n(\alpha)$ to optimize the upper bound of $\beta_n(\alpha)$. As indicated in Section 2.1, $\beta_n(\alpha)$ is limited to $[0, 1]$

Step 3: Selection.

The selection probability of the chromosome.

Step 4: Crossover.

After selection, two chromosomes, the two chromosomes are randomly selected to perform crossover operation.

Given a random indicator of crossover C_I , if $C_I > C_GA$, crossover operation continues; otherwise, it ends. When $C_I > C_GA$, given a random crossover coefficient $\gamma(\alpha)$.

Step 5: Mutation.

After selection and crossover operations, a chromosome, C_a and one belief distribution of the chromosome are randomly selected to perform mutation operation. Given a random indicator of mutation M_I , if $M_I > M_GA$, crossover operation continues; otherwise, it ends. When $M_I > M_GA$, given a random mutation probability $\eta(\alpha)$, when $\eta(\alpha) > 0.5$, belief degree are mutated to be increased.

Step 6: Update. After the selection, crossover, and mutation operations are performed, the fitness values of all chromosomes are recalculated to update the best objective with the corresponding solution. Step 7: Termination. If N_t iterations are completed, the best objective with the corresponding solution is obtained as the lower bound or upper bound of $\beta_n(\alpha)$. Otherwise, go to Step 3.

4. Conclusion Remarks

Comparison for generating expected utilities When the aggregated IVBDs of alternatives are not wished to be obtained, the optimal expected utilities of alternatives are required to generate a solution to a MCDM problem with the aid of a decision rule. In this situation, we compare the accuracy and efficiency performances of the four evolutionary algorithms for generating the optimal expected utilities by using specified and general IVBDs.

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Conflict of interest statement

The authors declare no conflict of interest in preparing this article.

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