Sumerianz Journal of Scientific Research, 2019, Vol. 2, No. 4, pp. 55-62
ISSN(e): 2617-6955, ISSN(p): 2617-765X
Website: https://www.sumerianz.com

Original Article Open Access

Are Axioms and Inference Rules Sufficient to Find any True Statement in Science? Gödel, A Great Logician with Psychological Problems Gave the Answer

Sotirios Sakkopoulos
Department of Physics, University of Patras, 265 00 Patras, Greece

Abstract

Until 1931 eminent intellectuals, like Russell, Ackermann and Hilbert were trying to find formal systems based of a finite number of axioms and generally accepted rules of inference in which every statement could be proved or disproved, independently from the objects considered. However, to apply this in an endless variety of objects they had to consider them as mere signs drained from any content. This had as a result a statement to be proved or disproved only syntactically, i.e. only formally, without caring if is true or not. Kurt Gödel in 1930 made a distinction between syntactic provability and semantic truth, the latter resulting from the consideration of properties coming from the content of the objects. Finally, in 1931 Gödel, with his two incompleteness theorems, proved that there are true statements in mathematics including elementary arithmetic of natural numbers [0,1,2…] which are impossible to be proved or disproved, demolishing the optimistic opinion prevailing until then. The consequences of these theorems puzzle logicians, mathematicians and physicists ever since. Although Gödel was a leading logician, ironically he was tormented by lifelong illogical fears which finally caused his death.

Keywords: Axiomatic foundation of science; Vienna circle; Formal system; Consistency; Completeness; Platonism in mathematics.

1. Introduction

Statements in Mathematics and Logic follow through generally accepted plausible rules starting from a small number of axioms, self – evident propositions derived logically from the idealization of the concepts included in the area. A crucial question is how one chooses the axioms. In non experimental sciences, like Mathematics and Logic the only way for this is through intuition [1].

Euclid of Alexandria (323 – 285 B.C.) was the first who succeeded to establish every theorem in Geometry on just five axioms. Until the nineteenth century this method remained indisputable, making scientist to wonder if not only Geometry but Mathematics and Logic or more generally Science as a whole could be founded on only a few axioms. Eminent Philosophers and Mathematicians like Bernard Russell, David Hilbert and Wilhelm Ackermann were optimistic that such a system of axioms was possible to be found.

Although the axiomatic form of Geometry appeared as the unshakable ideal example of scientific knowledge, drawbacks of it were discovered very early from the ancients. Particularly, the intricacy of the fifth axiom according to which through a point outside a given straight line only one parallel to the line intersecting it at infinity can be drawn in the plain defined by the point and the initial line, made ancient geometers to considered it as non self-evident and tried in vain to prove it from the other four axioms. Besides, ancients knew that apart from a straight line, there are curves like a hyperbola, which from a given point extents asymptotically to its axes meeting them at infinity [2, 3].

It was only in the nineteenth century when the impossibility of deducing the fifth axiom from the other four was proved and two non-Euclidean geometries appeared based on the first four of Euclid's axioms, each using its own version of the fifth. The hyperbolic geometry (Lobachevsky – Bolyai – Gauss geometry) in which it is assumed that more than one parallel can be drawn and the elliptic one (Riemannian geometry) in which no parallels can be drawn.

Apart from the fifth axiom, another weakness of the Euclidian method was confirmed. In 1882, Moritz Pasch gave an example of an obvious property of points and lines that could not be proved from Euclid's axioms. If the points A,B,C and D lie on a straight line and B lies between A and C and C lies between B and D then it is impossible to prove that B lies between A and D from Euclid's axioms [4].

The impossibility of proving certain true propositions within a given system of axioms is an undisputable weakness of this way of organizing knowledge. This deficiency was demonstrated by the Gödel's two theorems, namely that certain true propositions in the Arithmetic of natural numbers [0,1,2,3,…] cannot be proved or disproved from the accepted system of axioms proposed in 1899 by the Italian mathematician Giuseppe Peano. How Gödel came to these theorems and how their possible consequences affect Physics and other parts of knowledge is exposed in the following.

2. Gödel's Formative Years

Kurt Friedrich Gödel was born on the 28 April 1906 in Brünn, then in Austria – Hungary (now Brno, Chech Republic) in a wealthy family. When he was six years a rheumatic fever made him to believe that his heart was affected, an unjustifiable fear that became an everyday worry for him ever since.

Kurt attended school in Brünn and he was especially good in mathematics and languages, nicknamed "Mr. Why" (der Herr warum) because of his endless questions about everything. He entered the University of Vienna in 1924 intending to become a theoretical physicist but in 1926 turned to Mathematics partly as a result of the Philipp Furtwängler's lectures on Number Theory [5].

At this time Gödel started to participate in the Vienna Circle, a group of liberal intellectuals, who met regularly every Thursday at the University of Vienna. Among them were philosophers like Moritz Schlick, Rudolf Carnap, Carl Hempel, Otto Neurath and Ludwig Wittgenstein, logicians and mathematicians, like Karl Menger and Hans Hahn who introduced Gödel to this Circle and later became his doctoral advisor.

The central subject of discussions in the Circle was the relations between Language, World and Science. The positivism of this group according to which the exclusive source of knowledge is sensory experience interpreted through reason had as its main concern the exclusion of Metaphysics from Science.

Wittgenstein, who attended occasionally the meetings of the Vienna Circle, although disagreed with many of the ideas expressed there, influenced many of its members. Wittgenstein maintained that any language, scientific, mathematical or otherwise, was not sufficient to capture the way the World really is Sigmund [6].

Gödel did not accept all of the Circle's views, but listening to different aspects formulated his own ideas and only occasionally expressed them openly for discussion. In particular Gödel disagreed with Carnap's view that mathematics was nothing more than "syntax of language" i.e. a whole of rules for constructing or transforming symbols, which lacked any deeper essence.

On the contrary Gödel, from his first acquaintance with Mathematics, has been inclined to the notion that numbers and generally all the mathematical concepts are not created by human mind or language but they have an autonomous existence independent from the physical world waiting to be discovered and conceived not by reason but by intuition [7, 8]. This metaphysical attitude, called Platonism, because resembles Plato's theory of Forms, was at that time popular among mathematicians. Although Gödel disagreed with the positivism of the Vienna Circle, the interesting speculations of the scholars there undoubtedly played a principal role to his intellectual evolution.

Gödel's interests moved from Number Theory to Philosophy and Mathematical Logic after taking part in a seminar by Moritz Schlick which studied Bernard Russell's book *Introduction to Mathematical Philosophy*. Mathematical Logic became for Gödel "a science prior to all others, which contains the ideas and principles underlying all sciences" [9]. Very soon Gödel started to doubt not only about the adequacy of language for expressing the whole of the truths in the World, as Wittgenstein had suggested, but even for explicating the relationships among numbers in simple arithmetic. These speculations destined to take a mathematical form in his two famous theorems, as we shall see later.

3. Two Opposite Attitudes: Mechanization of Logic vs. intuitionism, Hilbert vs. Brouwer

Logicians at that time were trying to find a program able to rewrite all Mathematics based on a system of finite number of axioms and a few rules of inference by which reason makes conclusions. The axioms and the rules of inference had to be the basis of a formal system, which in addition includes a finite set of symbols (sometimes called the alphabet) from which formulas are constructed (i.e. finite strings of symbols) and a "grammar", i.e. the set of rules which tells how well-formed formulas are constructed from the alphabet symbols. A theorem of the formal system is any statement which is obtainable by a series of applications of the inference rules starting from the axioms. This procedure constitutes the proof of this statement.

The formal system had to be consistent and complete. Consistent means that there is no statement such that both the statement and its negation are provable from the axioms. Complete means that from the axioms every logical statement is provable, so every statement is either satisfiable or refutable. A statement which neither it nor its negation are theorems of the formal system is called undecidable [10, 11].

In 1928, David Hilbert and Wilhelm Ackermann published the book *Grundzüge der theoretischen Logik* (*Principles of Mathematical Logic*) in which the problem of completeness was posed, i.e. the possibility to find a finite number of axioms of a formal system from which every statement that is true in all models of the system could be derived. At that time Ludwig Wittgenstein and brilliant mathematicians like Bernard Russell, David Hilbert, and Wilhelm Ackermann were optimistic that such a system of axioms was possible to be found. Hilbert expected that the axiomatic formalization of Mathematics could be extended in a way to include sciences like Physics, in which Applied Mathematics played a fundamental role.

It was expected that from such a formal system all the statement of Mathematics could be deduced, independently from the objects considered, as natural numbers, real numbers, permutations, partitions, matrices, functions, groups, rings, fields, hexagons, points, lines, triangles, circles, spheres, polyhedra, topological spaces, manifolds etc. In Hilbert's words, "it does not matter if we call the objects chairs, tables, beer mugs or points, lines or planes". It is obvious that for such a system to be able to apply in this apparently unlimited variety of objects, it should be consisted from mere signs and sign configurations, drained from any particular property coming from their content. This strictly syntactic dealing with Mathematics came from Hilbert's belief that human mind was working by this way. His conviction was that "in the beginning is the sign", i.e. that the sign is the basic element of any mathematical or logical construction [12]. Hilbert's strict "mechanized" scheme undoubtedly appeared inadequate for "Platonist" Gödel.

The main opponent of Hilbert and its "mechanization" of Mathematics was the "heretic" Dutch Mathematician and Philosopher L.E.J. Brouwer, who characterized himself as "intuitionist". According to him Mathematics is a purely intellectual activity, in which the human mind creates first the mathematical "object", not just an empty sign, then the language is called to describe it and next the Logic intervenes. Logic is necessary for the language not for the object, which is already installed in human mind without language and Logic. Mathematics do not obey Logic, Logic obeys Mathematics [13]. In addition Brouwer speculated about how one chooses the axioms and the rules of inference.

In his book [14] Brouwer expresses his doubts on the value of Hilbert's mechanization of Mathematics. In such a mathematical system even if the language used ensured no contradiction, the value of a proof would be limited. It would provide something about the mathematical language itself, but not about mathematics in their essence. Objecting Hilbert's consistency proof, Brouwer stresses that this consistency of a mathematical system has to do with a language manipulating signs, not mathematical objects with content and properties [15].

Hilbert's mechanization of Mathematics on the one hand and Brouwer's intuitionism on the other constitute two extremes: mathematical objects are not just signs and sign configurations without content. They are meaningful entities and their properties come from their meaning. However, for the mentally conception of these mathematical objects a language oral or written is needed [16].

Gödel in 1928 attended Hilbert's lecture "*Problems in laying the foundations of mathematics*" at the International Congress of Mathematicians in Bologna concerning the consistency and completeness of mathematical systems. Moreover, in the same year Gödel might attended, or at least had a first – hand report about two lectures by Brouwer given in Vienna. In the first of them Brouwer, arguing against Hilbert, made a distinction between "consistent" theories and "correct" ones, the first constituting a larger part of Mathematics than the second. These ideas made Gödel to doubt if just the consistency of a formal system ensures its completeness as well. He suspected that apart from the statements and their corresponding negations which are the one provable and the other refutable, there may be some statements and their negations which are neither provable nor refutable (i.e. undecidable) in the system, once we have, as he says, "precisely stated formal means of inference" [17].

4. Semantic Truth vs. Syntactic Provability. Completeness vs. Incompleteness

Gödel started working on his doctoral thesis under Hans Hahn's supervision, sometime in 1928 or early 1929. The object of his dissertation was to prove the completeness of formal systems based on the axioms and the rules of inference given in Whitehead and Russell's *Principia Mathematica* and Hilbert and Ackermann's *Principles of Mathematical Logic*.

Gödel from the beginning realized the difference between semantic truth and syntactic provability and with his completeness theorem establishes a distinction between them. In his doctoral thesis Gödel proved the consistency and completeness of a semantic formal system, in the language of which, each term represents an object not an empty sign and so a semantic meaning is provided to the terms and formulas of this language.

It must be taken into account that a formal system is a cluster of models, (e.g. groups, fields, graphs, universes of set theory etc.), that satisfy the axioms and the rules of inference. Gödel's completeness theorem states that from a set of axioms and the principles of inference applied on them it is possible to generate all the true statements expressible in all the models of the system for the semantic entities which constitute it. In this sense the deductive system is characterized as "complete". Any statement that is unprovable from the set of axioms must actually be false in some model of these axioms. More specifically, Gödel in 1930 proved that any statement A is either provable in all models of the formal system considered, or A is false in the domain [0,1,2,3…] of the natural numbers, and therefore is not valid [18].

It must be stressed though that Gödel did not show the completeness of the arithmetic of natural numbers [0,1,2,3…] e.g. that every true statement about them could be proved. More generally any consistent formal system in which a certain amount of elementary arithmetic can be carried out, i.e. it contains Peano's axioms for natural numbers and all the symbols 0 "zero", S "successor of", +, ×, and =, is incomplete, as it was proved by Gödel in 1931, as we shall see later. At that time, theories of natural numbers and real numbers together were called "analysis", though theories of the natural numbers alone were known as "arithmetic" [19].

The elementary theory of natural numbers was considered as the most fundamental branch of Mathematics and the effort to formalize them should begin from number theory. At that time mathematicians believed that the set of Peano's axioms was complete, i.e. natural numbers constitute a complete model or, at worst, could be made complete by the addition of a finite number of axioms to the original list. In analogy to Euclid's axioms, the fifth of Peano's axioms, the principle of induction, proved to be an obstacle to the construction of such a complete set of axioms. It states: "Suppose that a property holds for 0, and suppose you can prove that if this property holds for another natural number, then it also holds for the successor of that number. Then the property holds for all natural numbers". Mathematicians thought it problematic because it does not talk about natural numbers themselves but about properties of them. Finally, the Norwegian logician Thoralf Skolem proved in 1934, three years after Gödel's incompleteness theorems, that it was impossible to replace this axiom with a finite number of more suitable propositions [20].

Gödel published his dissertation in 1930 and in the next year his famous two incompleteness theorems appeared in the 1931 volume of *Monatshefte für Mathematik* under the title: *Über formal unentscheidbare Sätze der Principia*

Mathematica und verwandter Systeme I ('On Formally Undecidable Propositions of Principia Mathematica and Related Systems I'). Several English translations of this paper which is considered a classic in mathematical logic can be found in print [21, 22].

5. The two Incompleteness Theorems. Gödel Numbers

The two incompleteness theorems state that:

- Any consistent formal system M within which a certain amount of elementary arithmetic can be carried out (natural numbers, Peano axioms and the symbols mentioned before are included) is incomplete with regard to statements of elementary arithmetic, i.e. there is at least one statement of the language of M about the natural numbers which can neither be proved nor disproved in M. This undecidable statement is often referred as "the Gödel sentence" it is denoted by the letter G and is in fact true.
- For any consistent formal system M within which a certain amount of elementary arithmetic can be carried out, the consistency of M cannot be proved in M itself, i.e. based on the axioms of M.

A schematic representation of the first incompleteness theorem for the formal system M, which is incomplete with regard to statements of elementary arithmetic is given by Casti and DePauli [5]. Presume a square which represents all possible statements that can be made about natural numbers. Initially, the entire square is gray. When a statement is proved true by the application of the rules of the formal system M, the colour of the associated logic space becomes white. When a statement is proved false the colour becomes black. The first incompleteness theorem says that always there is at least one statement like G, which is true but remains in the gray area, because it is undecidable, i.e. it is impossible to be proved or disproved from the axioms. The idea to overcome this obstacle by incorporating G into the axioms, as G is true and improvable like them, is not effective, because Gödel by his incompleteness theorem showed that in the new formal system there will be at least another statement undecidable like G.

An example of an arithmetic statement which is true but it is not possible to be derived from the axioms of arithmetic is Goldbach's theorem which states that every even number is the sum of two primes. Although this is true, no one has succeeded in finding a proof of it [2].

Let us see in a simplified way, what Gödel had proved in his 1931 paper.

First, Gödel had devised an ingenious way to assign a unique natural number, the "Gödel number", to each elementary sign, each formula and each proof. By this way, English-language phrases expressing statements are represented by formulas within arithmetic and the relations of logical dependence between statements are fully reflected in the numerical relations between their corresponding arithmetical formulas. For example the statement, "the sequence of formulas A, is a proof of the formula B", can be expressed as an arithmetical relation between the Gödel numbers of A and B. Thus mathematics can be mapped into arithmetic and to establish the truth or falsity of a mathematical statement, we need concern ourselves only with the question whether the relation between the Gödel numbers A and B holds. Conversely, we can establish that the arithmetical relation between a pair of Gödel numbers holds by showing that the mathematical statement mirrored by this relation is true [5].

By Gödel's arithmetization of mathematics the notion of provability within a system could be expressed purely in terms of arithmetical functions that operate on Gödel numbers of statements of the system. Therefore, the system which can prove certain facts about numbers can also indirectly prove facts about its statements. Questions about the provability of statements within the system are represented as questions about the arithmetical properties of numbers themselves.

Special concern was given to statements named by Hilbert "meta-mathematics", which refer to themselves and can lead to self-refuting results. An example is "this statement is false". If the statement is true, is false. And if the statement is false, it is true. Another example is, "the barber of Corfu shaves every man who does not shave himself. Who shaves the barber"? If he shaves himself, then he doesn't, and if he doesn't, then he does. Only logical systems which include arithmetic of natural numbers allow the encoding of statements about themselves to be made within their own language. That's why arithmetic plays a central role in the proof of Gödel's theorems.

To prove his two theorems Gödel followed these steps [2]:

He constructed an arithmetic formula G that represents the meta-mathematical statement: "The formula G is not provable". By Gödel's arithmetization the formula G is associated with a certain number h and it corresponds to the statement: "The formula with the associated number h is not provable".

Next Gödel showed that G is provable if, and only if, its formal negation is provable. This means that the arithmetic calculus is inconsistent. So, if the calculus is consistent, neither G not its negation is formally provable from the axioms of arithmetic. Therefore, if arithmetic is consistent, G is formally undecidable.

Gödel showed that, though G is not formally provable, it is a true arithmetic formula, in the sense that every integer satisfies it. This means that the axioms of_arithmetic are incomplete, i.e. there are arithmetical truths which cannot be deduced from the axioms.

Moreover, Gödel established that arithmetic is 'essentially' incomplete. This means that even if additional axioms were assumed so that the true formula G could be formally derived from the augmented set, another true but formally undecidable formula could be constructed.

Next, Gödel described how to construct an arithmetical formula A that represents the meta – mathematical statement: "Arithmetic is consistent". Next he proved that the formula representing the statement "if arithmetic is consistent (A) then there is G, a true and undecidable formula" is formally provable. Finally, he showed that the formula A is not provable. From this it follows that the consistency of arithmetic cannot be established by an argument that can be represented in the formal arithmetic calculus.

It may be confusing that Gödel in his doctoral thesis published in 1930 proved the completeness theorem and in 1931 proved the two incompleteness theorems. It must be stressed that there is a distinction between semantic completeness and syntactic completeness. A formal system can be semantically complete, as any statement which is impossible to be proved must be false in some model of the system, but containing "a certain amount of elementary arithmetic" becomes syntactically incomplete, since there are sentences about natural numbers, which although can neither proved nor disproved they are true. It must be noticed that Gödel in the original German he distinguished the two terms *vollständig* and *entscheidungsdefizit.* The first is translated as 'complete', but the second has the meaning of 'undecidable'. However, the term 'incomplete' has been prevailed in the English literature [12].

6. Comments on the two Incompleteness Theorems

Gödel's incomplete theorems demolished Hilbert's opinion that it is just a matter of time to find such a finite set of axioms that would allow "mechanically" either to prove or disprove, by proving its negation, every mathematical formula. However, Gödel did not prove that mathematicians cannot find solutions to any mathematical problem by pure reason, as they are not restricted to a given set of axioms. Any sentence undecidable in a certain formal system could be decidable in another one. Besides Stephen Hawking remarks:

The incompleteness theorem does not imply that every consistent formal system is incomplete, but it applies to those which are

1. Finitely specified, i.e. they have a finite number of axioms.
2. Large enough to include arithmetic for natural numbers.
3. Consistent

It is amazing that a truncated version of arithmetic, which does not have addition or multiplication, is complete. The latter is called Presburger arithmetic [23, 24]. The simultaneous presence of addition and multiplication makes the system incomplete. "Arithmetic" refers to individual numbers or terms. If a formal system does not have individual terms, like Euclidean geometry which makes statements about a continuum of points, circles and lines, it cannot satisfy second condition and it is complete. For the same reason the formal system of real numbers is complete.

It must be stressed that the undecidable true statements implied by the first incompleteness theorem is always relative to the formal system and refers to arithmetical statements. It does not demonstrate that there are truths that cannot be proved in general. This theorem does not apply where there is no formal system, like philosophical theories, religions, laws, constitutions, etc.

The concepts and methods introduced by Gödel in the incompleteness theorem make people speculate about the limits of computational procedures and the comparison between human mind and computers. Of course, computers are forced to use mechanically logical rules without resource to intuition. This makes people to accept that human mind surpasses the computers. However, it is a matter of debate of how much the human mind is similar to any finite computing machine. A question is if this sort of mind is consistent, but since humans make mistakes their minds are not consistent and so Gödel's theorems are not applicable.

7. Gödel's Incompleteness Theorem and Physics

Gödel's theorem that any consistent system which includes arithmetic contains true statements that it is not possible to be proven or disproven, made scientists and philosophers to extent its applicability far beyond Logic and Mathematics wondering if our view of the World and our quest to understand it are doomed to be always incomplete. In Quantum Mechanics natural numbers have a cardinal place. Moreover, Physics is based on experimental data from measurements which make use of Mathematics that ultimately rests on plain arithmetic of natural numbers, as the continuum of real numbers cannot be expressed practically.

The incompleteness theorem does not say anything about the ability of the hypotheses and equations of Physics to describe everything of the World [23]. Nonetheless, a physicist can never be sure that from the experimental data at his disposal is possible to prove or disprove every true result from them to formulate or modify the accepted hypotheses and theories. Freeman Dyson wrote: "The laws of Physics are a finite set of rules, and include the rules for doing Mathematics, so that Gödel's theorem applies to them". Besides Stephen [25]: "According to positivist philosophy of Science, a physical theory, is a mathematical model. So, if there are mathematical results that cannot be proved, there are physical problems that cannot be predicted" [26].

Scientists were divided in pessimists and optimists concerning this uncertainty. H. Weyl, considered Gödel's theorem as a statement discouraging research in Mathematics, as there are always true theorems escaping the effort of mathematicians to reach them from the axioms of a consistent formal system. The theologian and physicist S.L. Jaki states that Gödel's theorem prevents cosmologists to achieve a "theory of everything or final theory" which can explain and link together all physical aspects of the Universe [27, 28]. Although this seems disappointing, it has a positive side. The inexhaustible unanswered questions in Mathematics and Physics ensure that always will be a challenge for never – ending investigation and new discoveries for researchers. Gödel being a Platonist could not see any reason why we should have greater confidence in sense perception than in the mathematical intuition, which can lead us in the future to new physical theories agreeing with sense perceptions and at the same time questions not decidable now to acquire meaning and be decided eventually.

As it turns out from his later philosophical speculations, Gödel thought that by proving the incompleteness theorems, which showed that there are unprovable mathematical truths, demonstrated that there are mathematical objects, such as numbers, entities outside space and time not created by human mind, but awaiting to be discovered and conceived only by mind's eye, intuition [7].

8. From Vienna to Princeton

The publication of the two incompleteness theorems made Gödel internationally known and in 1933 and 1934 he gave a series of lectures at the Institute for Advanced Study (IAS) in Princeton, New Jersey entitle *On undecidable propositions of formal mathematical systems*. There Gödel met Albert Einstein and their friendship would last until Einstein's death in 1955. Gödel visited the IAS again in 1935, but the traveling and the hard work had as a result a nervous breakdown, as soon as he returned to Europe. He spent several weeks in a sanatorium recovering from depression. In spite from his problems Gödel proved important results on the consistency of the axiom of choice with the other axioms of set theory and developed an interest in Physics studying works of Eddington, Planck, Mach, Born, Lorentz, Dirac et al [12].

Hitler's rise to power in 1933 in Germany was the beginning of trouble for Gödel, although he was indifferent in politics. The political situation in Austria was deteriorated and the Vienna Circle was dissolved, as many of its members were Jews and all of them liberals suspicious to the Nazi regime. Many of them found refuse in the USA and the United Kingdom. Moritz Schlick, whose seminar prompted Gödel's interest in Logic was shot dead in June 22, 1936 by a former student [29]. This caused a severe nervous breakdown in Gödel, who developed paranoid symptoms, fearing of being poisoned. On 12 March 1938 Nazi Germany occupied Austria and Gödel could not find a position in the University of Vienna because of his association with the Vienna Circle.

On 20 September 1938 Gödel married Adele Nimbursky. Although Adele had little formal education, she was an extraordinarily intelligent person and played an extremely important role in Gödel's life. She stayed by him for the rest of his life, enduring his health and mental problems finding ways to make him to eat during periods of paranoid fear that someone was trying to poison him.

In September 1939 the World War II broke up and Gödel fearing conscription into the German army decided to flee Vienna with his wife for Princeton, which became Gödel's home for the rest of his life.

Resuming his mathematical work he published in 1940 the paper *The Consistency of the Axioms of Choice and Generalized Continuum Hypothesis with the Axioms of Set Theory*. Princeton: Princeton University Press. This work is a classic of Modern Mathematics. Gödel gradually abandoned his direct work in Mathematical Logic and turned towards Philosophy and Cosmology.

Einstein and Gödel were connected to each other with a very high esteem. Einstein's Theory of Relativity was a revolution for our notions about the Physical World and Gödel's incompleteness theorems had revealed an unexpected weakness of the abstract world of Mathematics. Both men regarded the World as something independent of our minds rationally organized and open to human understanding. However, they mistrusted the Quantum Theory and Gödel was skeptical about a Theory of Everything.

Gödel's acceptance that the World is rationally constructed and has meaning made him to believe in the afterlife and in a God not suggested by any religion, but by logic. Gödel constructed an ontological proof for God's existence around 1941, though he did not tell about this work until 1970, when he thought he was dying [30].

Gödel admired the polymath Gottfried Leibniz and he was especially interested in his *characteristica universalis*, i.e. a universal language capable to express any scientific concept based on the following strange idea: basic concepts would be encoded as prime numbers and more complicated ideas would be encoded by multiplying these numbers. So, if one wants to see which basic concepts are contained in a complicated idea he just had to break its characteristic number into a product of its prime numbers. This resembles the Gödel numbering used to encode statements of Logic as numbers. The super optimistic idea of Leibniz was very soon abandoned and a detailed treatment of that language was absent from Leibniz publications. This made Gödel excessively suspicious that there was a conspiracy against characteristica universalis and the publishers refused to communicate this language to the public or worse that the relevant works of Leibniz were deliberately destroyed [12]. In a lesser extent Gödel was involved with Immanuel Kant and Edmund Husserl.

Another concept that haunted Gödel's mind was time. He believed that time did not exist al all. He was not satisfied with philosophers' opinions. What he wanted was a rigorous proof for his opinion based on Mathematics. His association with Einstein and his ideas prompted Gödel to try to find a solution within Relativity Theory. The equations of General Relativity could provide different kinds of solutions. Einstein taking for granted on philosophical grounds that the Universe was eternal and unchanging, had "repaired" these equations to give such a Universe. On the other hand Georges Lemaître, a Belgian Catholic Priest, astronomer and professor of Physics, applying the same equations to Cosmology had proposed in 1927 an evolutionary approach of an expanding universe, which became known as the Big Bang Theory.

Gödel came up with another sort of solutions, according to which the universe was rotating and its geometry mixes up space and time in a way that one could travel back to any moment in his own past. Gödel was pleased with such a model: if one can visit his past then it has not really passed. Past, present and future are not different and so time has no meaning. Gödel offered these solutions to Einstein as a present for his seventieth birthday in 1949. However, instead to be pleased, Einstein was disturbed by Gödel's universe and started to have doubts about his own theory [31].

As age progressed, Gödel's physical and mental health deteriorated. He was constantly underweight, worried about his stomach and bowels condition, taking medicines of his own choice and distrusting physicians.

After Einstein's death on April 18th 1955 Gödel withdrew more and more from the world around him. Psychotic crises became chronic as time passed. The situation became extremely difficult as his wife's health was crumbling as well. Finally, in July 1977 his wife Adele was hospitalized and remained about five months away from Gödel. This combined with the Oskar Morgenster's death, who was a close friend of him, contributed to the steady decline of Gödel's health. A next door neighbor, some people from IAS and his close collaborator Hao Wang, tried in vain to help, as they were faced with his paranoid behaviour and his constant fear that there was a conspiracy to poison him. Finally Adele return home about the end of December 1977 and with a great effort persuaded him to enter the hospital. However, his prolonged starvation had affected irreversibly his health. On the 14^{th} January 1978 Gödel died of "malnutrition and inanition" resulting from "personality disturbance", as his death certificate reported. He weighted approximately 30 kilos. He was buried in Princeton Cemetery. Adele died in 1981 and was buried by him.

9. Some of Gödel's Quotes

Gödel's spirit could penetrate far beyond Mathematics and Logic expressing crucial remarks for many intellectual topics, as it is obvious from some of his quotes [32]:

"The more I think about language, the more it amazes me that people ever understand each other at all". This may reflect the speculations discussed in the Vienna Circle.

"I am convinced of the afterlife, independent of theology. If the world is rationally constructed, there must be an afterlife".

"Ninety percent of [contemporary philosophers] see their principle task as that of beating religion out of men's heads. ... We are far from being able to provide scientific basis for the theological world view".

"The formation in geological time of the human body by the laws of Physics (or any other law of similar nature), starting from a random distribution of elementary particles and the field is as unlikely as the separation of the atmosphere into its components. The complexity of the living things has to be present within the material [from they are derived] or in the laws [governing their formation]".

"I don't think the brain came in the Darwinian manner. In fact, it is disprovable. Simple mechanism can't yield the brain. I think the basic elements of the universe are simple. Life force is a primitive element of the universe and it obeys certain laws of action. These laws are not simple, and they are not mechanical".

"The development of Mathematics towards greater precision has led, as is well known, to the formation of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules…One might therefore conjecture that these axioms and rules of inference are sufficient to decide any mathematical question that can at all be formally expressed in these systems. It will be shown below that this is not the case that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers that cannot be decided on the basis of the axioms" (incompleteness theorems).

"Nothing new has been done in Logic since Aristotle".

"In principle, we can know all of Mathematics. It is given to us in its entirety and does not change. … That part of it of which we have a perfect view seems beautiful, suggesting harmony; that is that all the parts fit together although we see fragments of them only. … Mathematics is applied to the real world and has proved fruitful. This suggests that the mathematical parts and the empirical parts are in harmony and the real world is also beautiful".

Rudy Rucker, a mathematician with philosophical interests, gives a charming image of Gödel's personality far from the reserved, introvert person one could imagine [33], "Gödel certainly impressed me as a man who had freed himself from the mundane struggle. I visited him in the Institute office three times in 1972, and if there is one single thing I remember most, it is his laughter. … The conversation and laughter of Gödel were almost hypnotic. Listening to him I would be filled with the feeling of perfect understanding. He, for his part, was able to follow any of my chains of reasoning to its end almost as soon as I had begun it. What with his strangely informative laughter and his practically instantaneous grasp of what I was saying, a conversation with Gödel was very much like direct telepathic communication. Despite his vast knowledge, he still could discuss ideas with the zest and openness of a young man. If I happened to say something particularly stupid or naive, his response was not mockery, but rather an amused astonishment that anyone could think such a thing".

It is very interesting to refer to something said about Gödel, which must make us think very carefully about the value of human life [33], "Toward the end of his life, Gödel feared that he was being poisoned, and he starved himself to death. His theorem is one of the most extraordinary results in Mathematics, or in any intellectual field in this (20^{th}) century. If ever potential mental instability is detectable by genetic analysis, an embryo of someone with Kurt Gödel's gifts might be aborted".

10. Conclusion

Kurt Gödel was born with an unusually inquisitive mind, which was influenced by the intellectuals of the famed Vienna Circle. Gödel gradually formed his own beliefs disagreeing with eminent mathematicians and logicians of his time, like Hilbert, Ackermann and Russell. The latter were trying to find a suitably chosen finite set of obvious truths, the axioms and the proper rules of inference which could prove or disprove any statement in a consistent formal system. However, they considered mathematical objects syntactically, i.e. as signs, symbols without essence. Gödel had a different attitude. He believed that mathematical objects were not just signs, but had an objective existence beyond the physical world and one can reach them not only by reason, but by intuition as well, an approach called Platonism. By this way, mathematical objects are considered semantically, i.e. as meaningful entities with properties originated from their content. Gödel clearly stressed the difference between syntactic provability and semantic truth in his dissertation in 1930 and in the next year published the two famous incompleteness theorems. By them it was proven that in a formal system which contains the elementary arithmetic of natural numbers there are statements about them which is impossible to be proved or disproved from the axioms, although they are true. So, the optimistic opinion of Hilbert, Ackermann and Russell was demolished and a more general speculation was arisen if there are truths in Science especially in Physics that cannot be proven. Gödel is one of the most important logicians in history, but ironically he was tormented from his childhood by groundless fears about his health and as the age progressed, apart from this, he started to believe that somebody wanted to poison him. This fear led him to a pitiful death from “malnutrition and inanition”, as doctors certificated.

References

[1] Cory, L., 1997. "David Hilbert and the axiomatization of Physics (1894-1905)." *Archive for History of Exact Sciences*, vol. 51, pp. 83-198.

[2] Nagel, E. and Newman, J. R., 1958. *Gödel’s proof*. New York: University Press.

[3] Thomas, A. C. M., 2013. *Hyperbolic geometry and distance functions on discrete groups Thesis for the degree of Bachelor of Science with honours*. The University of New South Wales.

[4] Barrow, J. D., 2006. *Gödel and Physics, Paper delivered at: Symposium on horizons on truth Gödel Centenary 2006*. University of Vienna.

[5] Casti, J. L. and DePauli, W., 2000. *Gödel, a life of logic*. Cambridge: Perseus Publishing.

[6] Sigmund, K., 2017. *Exact thinking in demented times (the Vienna Circle and the epic quest for the foundations of Science*. New York: Basic Books.

[7] da Costa, N. C. A., 2011. "Gödel’s incompleteness theorems and Physics." *Principia*, vol. 5, pp. 453-459.

[8] Tait, W., 2005. *Truth and proof: The platonism in Mathematics. 3rd chapter in the book: The provenance of pure reason*. Oxford University Press.

[9] Gleick, J., 2011. *The Information: a history, a theory, a flood*. Pantheon Books.

[10] Kleene, S. C., 1967. *Mathematical logic*. New York: John Wiley & Sons, Inc.

[11] Franzén, T., 2005. *Gödel’s theorem. An incomplete guide to its use and abuse*. Wellesley, Massachusetts: A. K. Peters Ltd.

[12] Dawson Jr, J. W., 1997. *Logical dilemmas, The life and work of Kurt Gödel*. Wellesley Massachusetts: A.K. Peters Ltd.

[13] van Atten, M., 2017. "Luitzsen Egbertus Jan Brouwer, In E.N. Zalma (ed.)." *The Stanford Encyclopedia of Philosophy*, Available: http://plato.stanford.edu/cgi-bin/encyclopedia/archinfo.cgi?entry=brouwer

[14] Brouwer, L. E. J., 1996. "Life, art and mysticism. (translation by walter p. Van stigt)." *Notre Dame J. Formal Logic,*, vol. 37, pp. 389-429.

[15] Hesseling, D., 2012. *Gnomes in the fog: The reception of Brouwer’s intuitionism in the 1920s*. Basel: Birkhäuser-Springer.

[16] Martin-Löf, P., 2008. "The Hilbert-Brouwer controversy resolved? In the book, One Hundred Years of Intuitionism (1907-2007)." In *The Cerisy Conferences, eds. Mark van Atten, Pascal Boldini, Michel Bordeau, Gerhard Heinzmann. Birkhäuser-Verlag A.G.*

[17] Gödel, K., 1986. *Collected Works: Volume I: Publications 1929-1936* vol. 1. Oxford University Press.

[18] Kleene, S. C., 1987. *Kurt gödel 1906-1978*. Washington DC: Biographical Memoir, National Academy of Sciences.

[19] Heijenoort, V. J., 1967. *From Frege to Gödel: A source book in Mathematical logic, 1879-1931*. Harvard University Press.

[20] Oberdan, T., 1992. "The concept of truth in Carnap’s logical syntax of language." *Synthese*, vol. 93, pp. 239-260.

[21] Panu, R., 2015. "Gödel’s incompleteness theorems, In E. N. Zalma (ed.)." *The Stanford Encyclopedia of Philosophy*, Available: http://plato.stanford.edu/entries/goedel-incompleteness

[22] Gödel, K., 1992. *On formally undecidable propositions of principia mathematica and related systems translation by*. New York: B. Meltzer and Introduction by R. B. Braithwaite.

[23] Presburger, M., 1929. "About the completeness of a certain system of integer arithmetic in which addition is the only operation. , Warsaw 1929. Translation by Ryan Stansifer under the title “Presburger’s Article on Integer Arithmetic: Remarks and Translation” 1984." In *Paper presented by Presburger at the First Congress of Mathematicians of the Slavic countries.*Cornell University.

[24] Translation by Ryan Stansifer, 1984. *Under the title presburger’s article on integer arithmetic: Remarks and translation*. Cornell University.

[25] Hawking, S. "Gödel and the end of physics." Available: http://www.damtp.cam.ac.uk/events/strings02/dirac/hawking/

[26] Federman, S., 2006. *The nature and significance of Gödel’s incompleteness theorems*. Institute for Advanced Study, Princeton, Gödel Centenary Program.

[27] Jaki, S. L., 2004. "A late awaking to Gödel in Physics." *Sensus Communis*, vol. 5, pp. 153-162.

[28] Jaki, S. L., 2006. *On a discovery about Gödel’s incompleteness theorem*. Vatican City: Paths of Discovery. Pontifical Academy of Sciences Acta 18.

[29] Schlick Moritz. Available: https://en.wikipedia.org/wiki/Moritz_Schlick

[30] Oppy, G., 1996. "Godelian ontological arguments." *Analysis*, vol. 56, pp. 226-230.

[31] Ule, A., 2006. "Einstein, Gödel and the disappearance of time." *Synthesis Philosophica*, vol. 42, pp. 223-231.

[32] Gödel Quotations. Available: www.kevincarmody.com/math/goedel.html

[33] Kurt Gödel’s Quotes and Sayings. Available: https://www.inspiringquotes.us/author/1148-kurt-goedel/page:2