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Compton Effect and Transaspect Phenomenon

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Abstract

We study here the generalization of Compton scattering to co-particles, conceived as particles with negative energy in the framework of Pseudotachyonic Relativity. This includes co-electrons and co-photons (which are not the same as photons) and is a very important basis for the foundations of a field theory concerning either matter or co-matter, either both together. Analysing the statistical side of the problem, we come to discover the arising of an extraordinary "transaspect phenomenon", in certain cases of mixed energies: the pair of particles "*involved in the the collision*". instantly turn into their homologous co-particles.
Keywords: Special relativity; Pseudotachyonic relativity; Co-matter; Co-radiation; Compton effect; Compton scattering.

1. Introduction

1.1. Pseudotachyonic and Antibradyonic Relativity

In a recent paper [1], I reanalysed an alternative approach to fields of forces, mainly electrostatic and gravitational ones, depending on a balance of positive/negative energies. To achieve this goal, I begun by reviewing some basic ideas of Pseudotachyonic Relativity (PtR), the theory that specifically introduces time reversion and negative energies [2]. According to this theory, based on the standard *Lorentz pseudotachyonic transformations,*

$$\begin{cases} t^* = \frac{x/c-\beta t}{\alpha} \\ x^* = \frac{x\beta - ct}{\alpha} \\ y^* = y \\ z^* = z \end{cases} \text{ in which } \alpha = \sqrt{\beta^2 - 1}, \text{for } \beta = \frac{v}{c} > 1\,, \tag{1}$$

a **co-particle** is the only possible way to detect a tachyonic particle moving with velocity $v > c$; this co-particle moves with co-velocity

$$\hat{v} = \frac{c^2}{v} \tag{2}$$

and its energy and mass are given by

$$\hat{E} = -\frac{E_0}{\sqrt{1-(1/\beta)^2}} \quad \text{and} \quad \hat{m} = -\frac{m_0}{\sqrt{1-(1/\beta)^2}}. \tag{3}$$

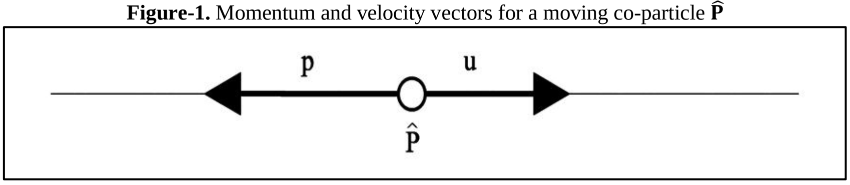
Intrinsically, this means that the proper energy and the proper mass of a *particle* **P** and of its *homologous co-particle* $\hat{\mathbf{P}}$ are exactly the same, in modulus.

Remark that the existence of tachyonic frames and particles is not possible unless it implicates a physical interchange of time and linear-coordinate axis [we will say that x and ict, as p_x and iE/c, are *connected variables*]. If not, even PtR transformations lead to inconsistencies. For instance, the co-velocity of $v = 0$ is an infinite one; so, since the De Broglie wave of a particle moving with velocity v propagates with phase velocity $u_\varphi = c^2/v$ (therefore detected as $\hat{u}_\varphi = v$), theoretically the wave of an immobile particle has an infinite velocity, along with an infinite wave length ($\lambda = u_\varphi/\nu_0$). Conversely, the transformations table (1) for $v \to \infty$ gives a pseudotachyonic immobile frame S^* in 'our' frame S:

$$\begin{cases} \lim\limits_{v\to\infty} t^* = \lim\limits_{v\to\infty} \frac{x/c-\beta t}{\sqrt{\beta^2-1}} = \lim\limits_{v\to\infty} \frac{x/(\beta c)-t}{\sqrt{1-1/\beta^2}} = -t \\ \lim\limits_{v\to\infty} x^* = \lim\limits_{v\to\infty} \frac{x.\beta-ct}{\sqrt{\beta^2-1}} = \lim\limits_{v\to\infty} \frac{x-ct/\beta}{\sqrt{1-1/\beta^2}} = x, \end{cases}$$

It is very important to note that co-particles, even massless ones (where $\hat{c} = c$), have a most remarkable dynamic characteristic: the velocity vector **u** and the corresponding momentum vector **p** have opposite orientations [see Figure 1]. Hence, *if we push a co-particle forward, it will go backwards*, and this is a crucial feature regarding the behaviour of co-particles in interactions.

Figure-1. Momentum and velocity vectors for a moving co-particle $\hat{\mathbf{P}}$



One may demonstrate that the electric charge is anti-invariant under PtR transformations [2], so, a particle and its homologous co-particle display opposite charges:

$$\hat{\mathbf{e}} = -\mathbf{e}. \qquad (4)$$

All this reminds the concept of *antiparticle*. And, indeed, in a first set of articles on PtR, [2-4], I identified co-matter with antimatter. But that are certain symptoms that co-particle and antiparticle – or, better saying, *prime-antiparticle* [this is, Dirac's negative-energy solution for his equation] – are not quite the same thing. In Luís [5], reviewd in Luís [1], I showed that this prime-antiparticle corresponds to an *antibradyonic Lorentz transformation* and I proposed that, together with its homologous particle, co-particle and co-prime-antiparticle, they are not but four **aspects** of a single entity, their **archeparticle** (or **matrix-particle**)[1]. A fundamental conclusion, then, is that there is no difference in nature between these aspects, the difference is nothing but a relativistic effect.

1.2. Compton Effect and Field Theory

As I proposed in Luís [4], Luís [5], Luís [1], the electrostatic or gravitational interactions between two particles concern the fields created by each one of them trough specific mediator particles: photons or co-photons in the first case, gravitons or co-gravitons in the second. This implies the emission by a material source of *particle* or *co-particle* mediators, which interact with target particles by exchanging energy and linear momentum. If, concerning a certain field, the mediator particles are **positive** (i.e. with positive energy), the field results **repulsive**; if the mediator particles are **negative** (co-particles, with negative energy), the field results **attractive**.

The electrostatic field created by an electron is a positive one, mediated by *photons*, which carry positive energy and a linear momentum with the same direction of its propagation; in electrostatic interactions, this momentum is (partially) transferred to another charged particle. If this particle is another electron, a *pro-reactive* one, it reacts moving away from the source; if it is a proton or a co-electron, both *anti-reactives*, they react (as co-particles ordinarily do) approaching the electron.

A proton produces a *negative field*, an attractive one, mediated by *co-photons*[2]; these co-photons carry negative energy and a linear momentum with the opposite direction of its propagation, that is to say, in the direction of their source; eventually, this momentum is (partially) transferred to another charged particle. An electron behaves reacting positively to the momentum received, approaching the proton; another proton or a co-electron, however, react negatively moving away from it.

So, we may conceive electrostatic interactions as being ruled by the Compton effect (generalized to co-particles). We must remember that, if so, mediator particles cannot simply disappear in the process, transferring all their energy and linear momentum to the target particle. I defend the hypothesis that gravitational interactions work alike. But then, *gravitons are co-particles* because their linear momentum must be opposite to their velocity, making the resulting field of a material particle to be attractive. On the other hand, the gravity field generated by co-matter is repulsive, due to positive co-gravitons. Naturally, matter is *pro-reactive* to gravity fields, co-matter *anti-reactive*; and all this creates the following *observable effects*:

- Two particles attract each other;
- Two co-particles also attract each other;
- A particle and a co-particle repel each other.

If these hypothesis are true, one can easily understand why it is so important, besides its intrinsic interest, to well understand the generalization of Compton effect to co-particles. That is why I return to the subject. And, as a gift, in the process, we will find out some astonishing new results.

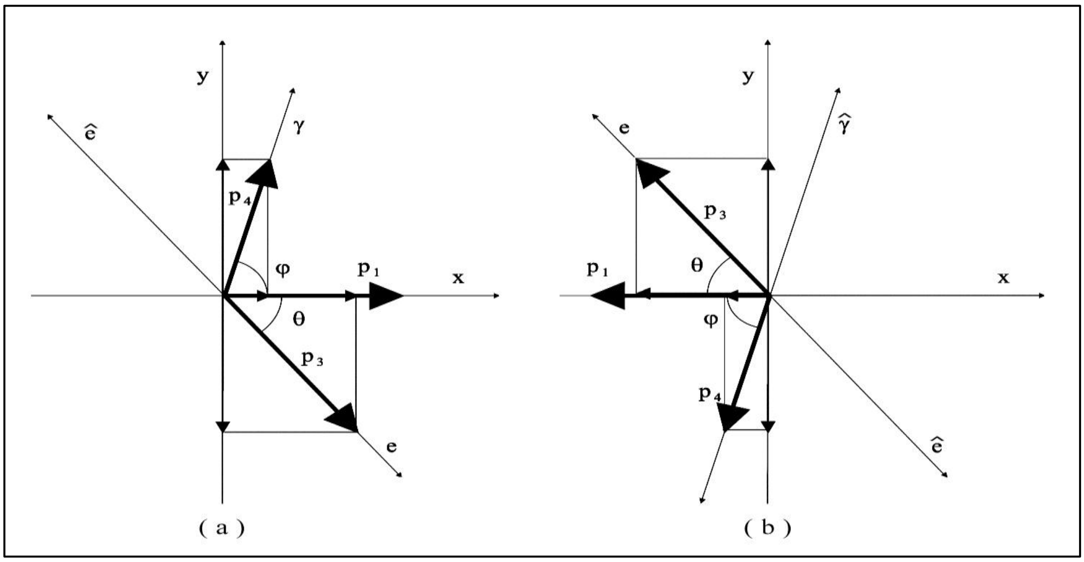
2. Compton Effect Basics

2.1. The Theory

We will start by reviewing the subject, mainly as exposed in Luís [3][3]. Classically, the Compton effect – or Comtpon scattering – concerns a photon (particle **1**) hitting an electron (particle **2**), supposed immobile in our reference frame. In the scattering process, there is a transference of energy and linear momentum, the subsequent states of the photon (now particle **3**) and the moving electron (particle **4**) being related by some precise equations. We may generalize this interaction to co-particles as follows (all the following equations are deduced in Appendix A).

Figure-2. Momentum vectors for: a) a photon colliding with an electron or a co-electron; and b) a co-photon colliding with an electron or a co-electron

[1] However, I also concluded, in an appendix of [1] that a brief analysis "seems to indicate that the two aspects, prime-antiparticle and co-particle, may indeed correspond to a single one, evaluated in a natural base and its dual base." This is yet an open issue
[2] Due to the inversion of time in PtR transformations, a *co-electron* should appear not emitting but absorbing co-photons; however, this does not change the result of the interaction.
[3] We will do it, replacing the word *"antiparticle"* by *"co-particle"*.



Let φ and θ be the angles the vectors $\mathbf{p_3}$ and $\mathbf{p_4}$ form with the vector $\mathbf{p_1}$ [see Figure 2], in the reference frame where $\mathbf{P_2}$ is immobile and the incident $\mathbf{P_1}$ moves along the x-axis in its positive sense. Then, because of the opposite orientations of velocity **u** and linear momentum **p** in the case of co-particles, the **scattering angles** (or displacement angles) for both particles – *relatively to the x-axis positive sense* – are given by

$$\begin{cases}\varphi_s = (-1)^{k_1}\cdot\varphi \\ \theta_s = k_1\cdot 180° + (-1)^{k_1}(k_2\cdot 180° + \theta)\,,\end{cases} \tag{5}$$

where

$$\begin{cases}k_i = 0 \quad \text{for particles; and} \\ k_i = 1 \quad \text{for co} - \text{particles.}\end{cases}$$

Making $E_1 = E$, $E_3 = E'$ and $E_2 = E_0$ (the rest energy of $\mathbf{P_2}$, this is, the electron or co-electron), the well known mathematical expression for the Compton effect is

$$\frac{1}{E\prime} - \frac{1}{E} = \frac{1}{E_0}(1 - cos\varphi). \tag{6}$$

Remark that this equation is valid either for particles or co-particles, either for incident photons or co-photons, simply because the signs of the variables energy or linear momentum have no relevance in its deduction. As noted before, it cannot be satisfied for $E' = 0$ and this means that the incident photon or co-electron can never be entirely absorbed by a free particle [6]. There is always a remaining energy for the scattered photon or co-photon.

Applying the quantum relations $E = \frac{h.c}{\lambda}$ and $E' = \frac{h.c}{\lambda\prime}$, we may re-write the expression for the Compton effect as (making $\Delta\lambda = \lambda' - \lambda$)

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\varphi), \tag{7}$$

in the condition that we consider for co-photons a *negative* wavelength (this means, contrary to its propagation velocity). This is the most commonly used form to mathematically transcribe the Compton effect. As indicated above, in the calculus, concerning co-photons, we must consider the angle

$$\varphi = -\varphi_s.$$

Because of this, making $p = E/c$ and $p' = E'/c$ for the incident and the scattered particles respectively (which makes p and p' *negative* for co-photons), one gets for the components of the momentum p':

$$\begin{cases}p'_x = p'\cos\varphi \\ p'_y = (-1)^{k_1}p'\sin\varphi,\end{cases} \quad \text{where} \quad p' = \frac{h}{\lambda\prime}, \tag{8}$$

and, consequently, for the momentum of the particle **4** set in motion:

$$\begin{cases}p_{4x} = p - p'_x = p - p'\cos\varphi \\ p_{4y} = -p'_y = (-1)^{k_1+1}p'\sin\varphi.\end{cases} \tag{9}$$

Other interesting equations are those obtained for the kinetic energy E_k, the factor $\beta = v/c$, the total momentum and the angle θ of the set in motion target particle, $\mathbf{P_4}$:

$$\begin{aligned}&E_k = -\Delta E = E - E' \quad \Rightarrow \quad E_k = hc\,\frac{\Delta\lambda}{\lambda\,\lambda\prime}\,; \\ &\beta^2 = 1 - \frac{E_0^2}{(E_0+E_k)^2} = \frac{E_4^2 - E_0^2}{E_4^2}\,; \\ &p_4 = \beta E_4/c \quad or \quad p_4 = \frac{(-1)^{k_2}}{c}\sqrt{E_k(E_k+E_0)}\,; \\ &\tan\theta = \frac{\lambda\sin\varphi}{\lambda\cos\varphi - \lambda\prime}.\end{aligned} \tag{10}$$

For $\varphi = 0°$, these equations conduce to $\Delta\lambda = 0$, $E_k = 0$ and an indetermination to $\tan\theta$ [4]; this means that the (co-)photon does not scatter and the (co-)electron remains still. However, $\varphi = 180°$ conduces to $\Delta\lambda = 2h/m_0c$,

[4] One may verify that $\lim\limits_{\varphi\to 0°}\theta \to \pm 90°$.

$E_k = 2h^2/m_0\lambda\lambda'$, its maximal value and $\tan\theta = 0$. In this case, an incident photon comes right back; an electron goes forward along the x-axis, whilst a co-electron moves in the opposite direction (and the same for an incident co-photon with a co-electron or an electron).

Concerning the angle θ, one may use the alternative Debye's formula (adapted with the - sign):

$$\cot\theta = -\left(1+\frac{h}{m_0 c\lambda}\right)\tan\frac{\varphi}{2}. \tag{11}$$

As I noted in [3], this equation clearly shows that, for an incident photon, which may be scattered with any angle $(0° \leq \varphi_s < 360°)$, an electron is confined within the space frontal region $(-90° < \theta < 90°)$. This means that the electron always moves forward. But, for an incident **co-photon**, since, for the **electron**, $\theta_s = 180° - \theta$, the hit particle moves with a scattering angle within the interval $90° < \theta < 270°$, thus confined to the anterior region; this is, it moves backwards! Another important feature is that, in this case, **the wavelength diminishes**, that is to say **the energy of the co-electron increases** (both in modulus). This negative energy increment counterbalance exactly the kinetic energy of the electron set in motion. The results are the same if we consider respectively an incident co-photon or a photon against a co-electron (the photon's energy also increases).

* It is important to note that this conclusions do not remain strictly valid for angles φ in the weird *"transaspect gap"* (its frontiers included), a subject we will analyse further ahead.

Meanwhile, remark that the Compton effect is perfectly reversible in time, since the conservation principles, for the energy and linear momentum implicit in it do not depend on the sense of the time arrow. This is physically legitimated by the observation of the *inverse process* in Nature [7] and theoretically by the fact that it corresponds to a simple pseudotachyonic transformation (for instance, the transformation of the collision $\hat{\gamma} \mapsto \hat{\mathbf{e}}$ is the inverse classical Compton effect ($\gamma \mapsto \mathbf{e}$); we will say these two are *time-equivalent processes*).

2.2. Some Examples

Allow me to illustrate this subject with four numerical examples.

1) Collision PHOTON ($\mathbf{P_1}$) $\mapsto$ ELECTRON ($\mathbf{P_2}$)

This is the classical Compton effect. Take, for instance, a photon in the gamut of the X-rays, with $\lambda = 1{,}00$ Å, and an angle $\varphi_s = 75°$. Since $k_1 = 0$, $\varphi = \varphi_s$ and we obtain

$\Delta\lambda = 0.0243\ \text{Å} \cdot (1-\cos 75°) = 0.0180\ \text{Å} \quad \Rightarrow \quad \lambda' = 1.1018\ \text{Å}$

and also, since $k_2 = 0$,

$E_k = 219.03\ \text{eV}$

$\theta = -51.83° \quad \Rightarrow \quad \theta_s = \theta = -51.83°,$

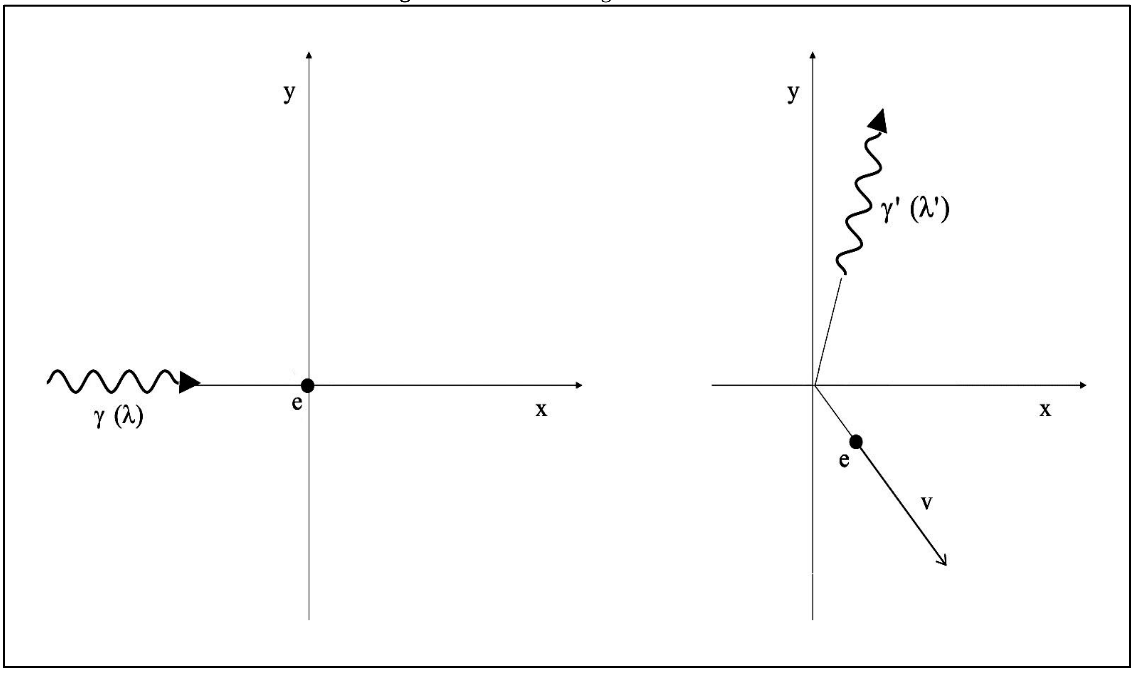
which expresses the real scattering angle of the hit electron [see figure 3]. The electron's velocity is

$v = 0.0293\, c = 8.77 \times 10^6\ \text{m/s}.$

For the linear momentum, $p = 6.63 \times 10^{-24}$; $p' = 6.51 \times 10^{-24}$ and $p_4 = 8.00 \times 10^{-24}$, in $\text{Kg}\cdot\text{m/s}$ units, with components:

$$\begin{cases} p'_x = 1.68 \times 10^{-24} \\ p'_y = 6.29 \times 10^{-24} \end{cases} \quad \text{and} \quad \begin{cases} p_{4x} = 4.94 \times 10^{-24} \\ p_{4y} = -6.29 \times 10^{-24} \end{cases} \quad \text{Kg}\cdot\text{m/s}.$$

Figure-3. Photon colliding with an electron



2) Collision CO-PHOTON ($\hat{\mathbf{P}}_1$) $\mapsto$ CO-ELECTRON ($\hat{\mathbf{P}}_2$)

For an incoming co-photon, with $\lambda = -1{,}00$ Å, considering that the mass m_0 of the co-electron is negative, we get for the same scattering angle $\varphi_s = 75°$, this is $(k_1 = 1)$ $\varphi = -75°$. So,

$\Delta\lambda = -0.0243\ \text{Å} \cdot (1-\cos 75°) = -0.0180\ \text{Å} \quad \Rightarrow \quad \lambda' = -1.1018\ \text{Å}$

and

$E_k = -219.03\ \text{eV},$

which are symmetrical values than those we obtained before; also,

$\theta = 51.83°.$

Since we are dealing with co-particles ($k_1 = k_2 = 1$), the co-electron scattering angle is [see Figure 4]:

$\theta_s = -\theta = -51.83°.$

The velocity of the co-electron is the same as the velocity of the precedent electron:

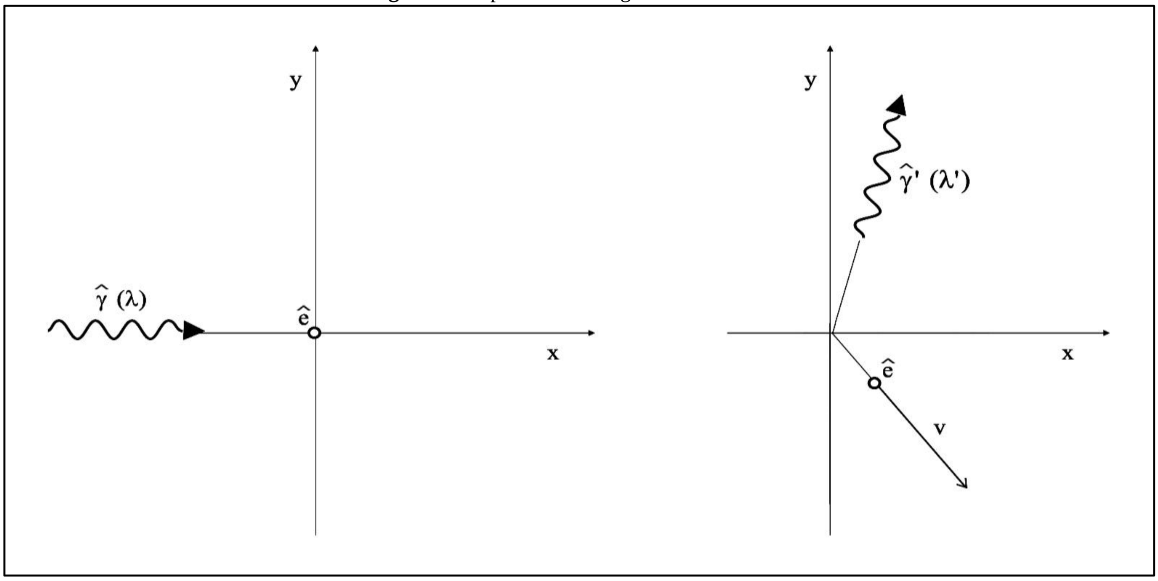
$$v = 0.0293\, c = 8.77 \times 10^6\ \text{m/s}.$$

For the linear momentum, $p' = -6.51 \times 10^{-24}$ and $p_4 = -8.00 \times 10^{-24}$ Kg · m/s, with components:

$$\begin{cases} p'_x = -1.68 \times 10^{-24} \\ p'_y = -6.29 \times 10^{-24} \end{cases} \quad \text{and} \quad \begin{cases} p_{4x} = -4.94 \times 10^{-24} \\ p_{4y} = 6.29 \times 10^{-24} \end{cases} \quad \text{Kg} \cdot \text{m/s}.$$

These results – as one should expect – correspond exactly to those obtained in the former example. They are compatible with those we should obtain from the pseudotachyonic transformation $S^* \to S$ (making S the paraframe of S^*) applied to the usual collision *photon* ↦ *electron* in the frame S^*. However it is not the same thing because there is an inversion of time implicit in the transformation; that's why we will say they are *time-equivalent processes*.

Figure-4. Co-photon colliding with a co-electron



3) Collision CO-PHOTON ($\hat{\mathbf{P}}_1$) ↦ ELECTRON ($\mathbf{P}_2$)

For an incoming co-photon, with $\lambda = -1{,}00$ Å, considering that now the mass m_0 is positive, we obtain for the scattering angle $\varphi_s = 75°$ ($\varphi = -75°$)

$$\Delta\lambda = 0.0243\ \text{Å} \cdot (1 - \cos 75°) = 0.0180\ \text{Å} \quad \Rightarrow \quad \lambda' = -0.9820\ \text{Å}.$$

We see that, as noted before, **the wavelength diminish** in modulus, that is to say **the energy of the co-photon increases** (also in modulus). This negative energy increment is the exact counterbalance of the positive energy transmitted to the electron set in motion:

$$E_k = 227.07\ \text{eV}.$$

Remark that this value of E_k is slightly superior to the one resulting from the collision *photon* ↦ *electron*; it corresponds to an also slightly superior value for velocity,

$$v = 0.0298\, c = 8.93 \times 10^6\ \text{m/s},$$

and for the angle θ,

$$\theta = 53.18°.$$

In this case, $k_1 = 1$ and $k_2 = 0$; therefore, the scattering angle for the electron is

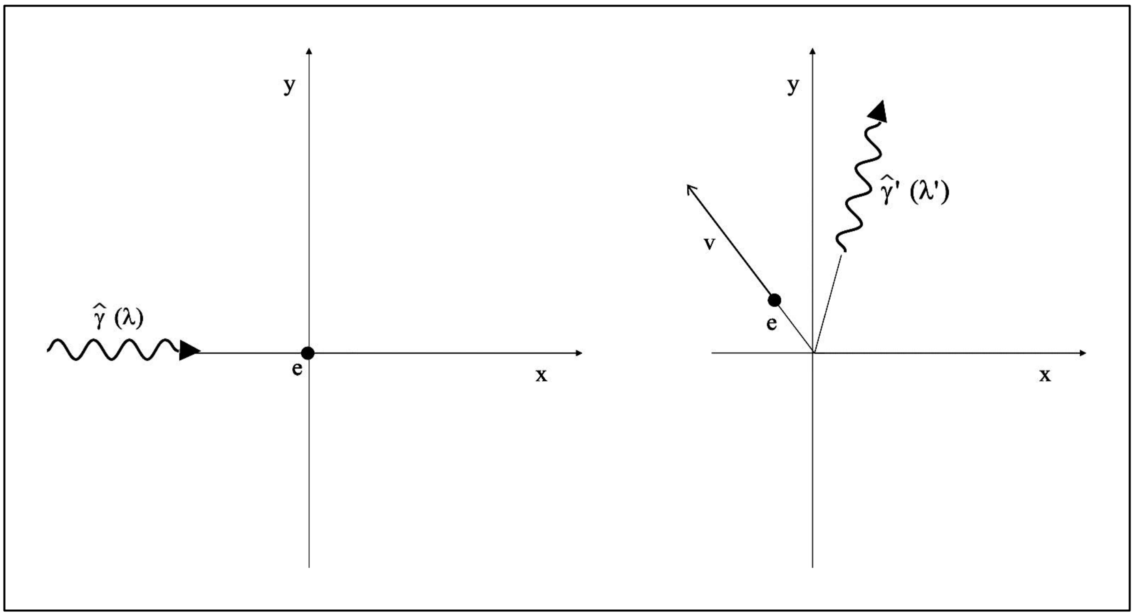
$$\theta_s = 180° - \theta = 126.82°,$$

meaning that **the electron goes back** towards the xx axis sense, and this in the same semi-space (either superior or inferior) where the scattered co-photon moves [see Figure 5]. This is a natural result in view of the negative linear momentum transferred from the co-photon to the electron.

For the linear momentum, $p' = -6.75 \times 10^{-24}$ and $p_4 = 8.14 \times 10^{-24}$ Kg · m/s, with components:

$$\begin{cases} p'_x = -1.75 \times 10^{-24} \\ p'_y = -6.52 \times 10^{-24} \end{cases} \quad \text{and} \quad \begin{cases} p_{4x} = -4.88 \times 10^{-24} \\ p_{4y} = 6.52 \times 10^{-24} \end{cases} \quad \text{Kg} \cdot \text{m/s}.$$

Figure-5. Co-photon colliding with an electron



4) Collision PHOTON ($\mathbf{P_1}$) ↦ CO-ELECTRON ($\mathbf{\hat{P}_2}$)

Finally, for an incoming photon, with $\lambda = 1{,}00$ Å, considering that once again the mass m_0 is negative, we obtain for the angle $\varphi_s = \varphi = 75°$:

$$\Delta\lambda = -0.0243 \text{ Å} \cdot (1 - \cos 75°) = -0.0180 \text{ Å} \quad \Rightarrow \quad \lambda' = 0.9820 \text{ Å}.$$

Like in the precedent situation, **the wavelength diminish** and **the energy of the photon increases.** This now positive energy increment counterbalance the negative kinetic energy of the co-electron set in motion, which is

$$E_k = -227.07 \text{ eV}.$$

The angle θ results identical:

$$\theta = -53.18°,$$

but in this case, $\mathbf{P_2}$ being a co-particle, $k_1 = 0$ and $k_2 = 1$; therefore, the scattering angle for the co-electron is:

$$\theta_s = 180° + \theta = 126.82°;$$

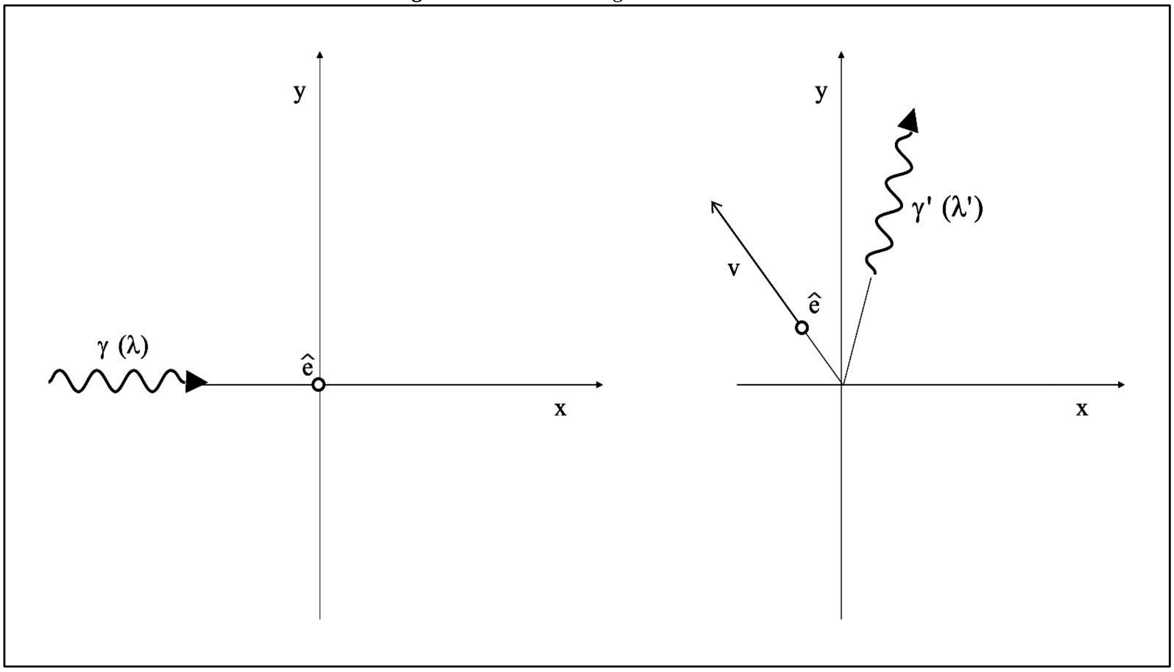
so, the final result is that the **co-electron goes back** [see Figure 6] with the same velocity calculated in 3) and once again in the same semi-space (either superior or inferior) of the scattered photon. The co-electron reacts negatively to the positive linear momentum transferred from the incident photon.

For the linear momentum, $p' = 6.75 \times 10^{-24}$ and $p_4 = -8.14 \times 10^{-24}$ Kg · m/s, with components:

$$\begin{cases} p'_x = 1.75 \times 10^{-24} \\ p'_y = 6.52 \times 10^{-24} \end{cases} \quad \text{and} \quad \begin{cases} p_{4x} = 4.88 \times 10^{-24} \\ p_{4y} = -6.52 \times 10^{-24} \end{cases} \quad \text{Kg} \cdot \text{m/s}.$$

Coherently, the collision *photon* ↦ *co-electron* may be obtained, as before, with an invertion of time, from the pseudotachyonic transformation $S^* \to S$ (making S the paraframe of S^*) applied to the collision *co-photon* ↦ *electron* in the frame S^*. As before, we have here two time-equivalent processes.

Figure-6. Photon colliding with a co-electron



3. Further Meditations on Compton Effect

All this picture is quite beautiful and self-satisfying. But now some disturbing clouds appear in the horizon.

There are no problems as long as we deal with pairs of (positive) particles (the classic Compton effect) or pairs of co-particles (because this case is time-equivalent to the first, signs for both particles changing concomitantly; in fact, this issue consists on a simple pseudotachyonic transformation of the inverse process). We will say that, in one case or the other, the particles of the couple are of a *correspondent aspect*. Problems arise when particles are of *opposite aspects*, when we deal with *mixed energies*, this is a co-photon hitting an electron or a photon hitting a co-electron.

If we wish to study the mechanics of a force field mediated by huge numbers of mediator particles, we will need to deal with *average values* for variables like energy or linear momentum. That is why, in field theory, I began studying this problem; and then I stumbled on a serious issue, a mysterious, incomprehensible and apparently condemning constraint concerning the average energy of the scattered incident particle and, therefore, of the scattered particle.

Let us take a look at the average values for variables connected with the Compton effect in the gap $[0,2\pi]$ for the angle φ. We will bear in mind that, for a function $y = f(x)$, its average value in the gap $[a,b]$ is given by

$$\bar{y}\ |_a^b = \frac{1}{b-a}\int_a^b y\ dx.$$

• For the wavelength shift,

$$\Delta\lambda = \lambda_0(1-cos\varphi), \quad \text{considering} \quad \lambda_0 = \frac{h}{m_0 c},$$

we obtain an average value in the gap $[0,2\pi]$:

$$\overline{\Delta\lambda}\Big|_0^{2\pi} = \frac{w}{2\pi}[\varphi - \sin\varphi]_0^{2\pi} \quad \Rightarrow \quad \overline{\Delta\lambda} = \lambda_0 = \frac{h}{m_0 c} = \frac{hc}{E_0}. \tag{12}$$

Remark that this average is independent from λ and that it corresponds to the wavelength shift for the angle $\varphi = \pm\frac{\pi}{2}$, the so-called *Compton wavelength*.

• Concerning the angle θ (for the linear momentum of the hit particle), using Debye's formula, we get a quite complicated equation:

$$\bar{\theta}\Big|_0^{2\pi} = -\frac{1}{2\pi}\int_0^{2\pi} \arctan\left[\frac{1}{\left(1+\frac{h}{m_0 c\lambda}\right)\tan\frac{\varphi}{2}}\right] d\varphi;$$

however, it is easy to conclude that, for symmetry reasons (for each $\theta \neq n \cdot 180°$, there is a $-\theta$), it must be

$$\bar{\theta} = 0°\ ; \quad and,\ consequently, \quad \overline{\theta_s} = [k_1 + (-1)^{k_1} k_2] \cdot 180°$$

for the scattering angle average; this is,

$$\begin{cases} 0° & for \quad \gamma \mapsto \mathbf{e} \quad or \quad \hat{\gamma} \mapsto \hat{\mathbf{e}}; 4pt \\ 180° & for \quad \gamma \mapsto \hat{\mathbf{e}} \quad or \quad \hat{\gamma} \mapsto \mathbf{e}. \end{cases}$$

I verified this result in several numerical calculus of discrete averages.

• For the energy E' of the scattered photon or co-photon, since, according to (9),

$$E' = \frac{E.E_0}{E_0 + E(1-\cos\varphi)}, \tag{13}$$

we obtain:

$$\overline{E'}\Big|_0^{2\pi} = \frac{E}{2\pi}\int_0^{2\pi}\frac{d\varphi}{1+E/E_0(1-cos\varphi)}.$$

I prove in the Appendix B that the algebraic solution of this equation is given by

$$\overline{E'} = E\sqrt{\frac{E_0}{E_0+2E}}. \tag{14}$$

Therefore, for the average of the kinetic energy, this is, the transferred energy to the target particle, $E_k = -\Delta E = E - E'$, yields

$$\overline{E_k} = E\left(1 - \sqrt{\frac{E_0}{E_0+2E}}\right). \tag{15}$$

• Finally, for the linear momentum of the target particle, as we have seen:

$$\begin{cases} p_{4x} = p - p'_x = p - p'\cos\varphi \\ p_{4y} = -p'_y = (-1)^{k_1+1}p'\sin\varphi. \end{cases}$$

The average $\overline{p}_{4y}$ for the orthogonal component in the y-axis must be null because all the positive values are symmetrically cancelled by negative ones. This means that what really matters is the average on the xx component of $\mathbf{p}_4$:

$$\overline{\mathbf{p}}_4 = \overline{\mathbf{p}}_{4x}.$$

Developing the equation $cp_{4x} = E - E'\cos\varphi$, using the expression (13) for E', we obtain

$$cp_{4x} = (E + E_0)\frac{E(1-\cos\varphi)}{E_0+E(1-\cos\varphi)};$$

but then, knowing that

$$E_k = -\Delta E = E - E' = \frac{E^2(1-\cos\varphi)}{E_0+E(1-\cos\varphi)},$$

we may write

$$cp_{4x} = \frac{E+E_0}{E}\ E_k \quad \text{and, so,} \quad \overline{p_4} = \overline{p_{4x}} = \frac{E+E_0}{c\ E}\ \overline{E_k}$$

or, finally,

$$\overline{p_4} = \overline{p_{4x}} = \frac{E+E_0}{c}\left(1 - \sqrt{\frac{E_0}{E_0+2E}}\right) \tag{16}$$

The formulae (15) and (16), for $\overline{E_k}$ and $\overline{p_4}$, are confirmed by numerical calculus of discrete averages. For instance, for an electron target ($E_0 = 8.20 \times 10^{-14}$ J) and an incident **photon** which wavelength is $\lambda = 1\text{Å}$ ($E = 1.99 \times 10^{-15}$ J) – the first example in the previous section – we obtain

$\overline{E_k} = 4.65 \times 10^{-17}$ J,

whilst the discrete average for all $\varphi = n \cdot 3°$, with $0 \leq n \leq 120$, symbolised by $\overline{[E_k]}$, is $\overline{[E_k]} = 4.64 \times 10^{-17}$ J.

For an incident **co-photon** with the same wavelength, in modulus ($\lambda = -1\text{Å}$) – the third example –, we get

$\overline{E_k} = 5.00 \times 10^{-17}$ J,

an energy slightly superior to the former one ($1.08\ \times$); this process is then a little more 'efficient'. A similar calculus for a discrete average gives $\overline{[E_k]} = 4.99 \times 10^{-17}$ J.

Concerning the momentum, we obtain respectively:

$\overline{p_4} = 6.55 \times 10^{-24}$ Kg · m/s for $\overline{[p_4]} = 6.53 \times 10^{-24}$ Kg · m/s

and

$\overline{p_4} = 6.71 \times 10^{-24}$ Kg · m/s for $\overline{[p_4]} = 6.69 \times 10^{-24}$ Kg · m/s.

However, it meets the eye that the related average formulae (14) to (16) implicate a curious constraint in order to avoid imaginary numbers:

$$\frac{E_0}{E_0+2E} \geq 0 \quad or \quad \begin{cases} \text{for } E_0 > 0 & \Rightarrow \quad E > -\frac{1}{2}E_0 \ ; \\ \text{for } E_0 < 0 & \Rightarrow \quad E < -\frac{1}{2}E_0. \end{cases} \tag{17}$$

This enigmatic constraint has been, for me, the first symptom that something odd was going on. How can we explain it if the corresponding *wavelength shift* is continuous and the *energy shift* also appears to be?

Well, to answer this and simplify equations, I'll introduce the ***K* factor**:

$$K = \frac{E}{E_0}, \tag{18}$$

in such a manner that:

- if $K > 0$, the two particles are of an identical *aspect*;
- if $K < 0$, the two particles are of opposite *aspects*.

It is easy to conclude that Lorentz transformations (including tachyonic and pseudotachyonic transformations) do not modify the sign of the K factor.

In these terms we may rewrite the precedent equations as

$$\overline{E'} = E\sqrt{\frac{1}{1+2K}} \tag{19}$$

and

$$\overline{E_k} = -\overline{\Delta E} = E\left(1 - \sqrt{\frac{1}{1+2K}}\right), \tag{20}$$

'normally' submitted to the constraint (17),

$$1 + 2K > 0 \quad \text{or} \quad K > -\frac{1}{2}, \tag{21}$$

and, finally, for the energy of the scattered co-particle:

$$E' = \frac{E}{1+K(1-\cos\varphi)}. \tag{22}$$

This last equation is quite surprising because, apparently, there is no discontinuity point for the Compton effect translated by the *wavelength shift* equation (7); however, the equation above has discontinuity points for

$$1 + K(1 - \cos\varphi_c) = 0 \quad \Rightarrow \quad \cos\varphi_c = \frac{1}{K} + 1. \tag{23}$$

We will name **critical angles** the two symmetrical φ_c resorting from this equation. Remark that, because $-1 \leq \cos\varphi_c \leq 1$, their existence is only possible for

$$-2 \leq \frac{1}{K} \leq 0 \quad \Rightarrow \quad K \leq -\frac{1}{2}, \tag{24}$$

this is, for mixed energies involved, *pairs of particles of opposite aspects*; there are no critical angles or discontinuity points concerning the classic Compton effect or its time-equivalent, a co-photon hitting a co-electron.

Therefore,

$$\lim_{\varphi\to\varphi_c} E' = \lim_{\varphi\to\varphi_c} \frac{E}{1+K(1-\cos\varphi)} = \pm\infty \quad \text{and, so,} \quad \lim_{\varphi\to\varphi_c} E_4 = \mp\infty,$$

according to the $\pm$ signs of the initial energy E.

But *what is the physical meaning of these discontinuity points*, apparently contradicting their non-existence in (7)? And what happens beyond the critical angles? We will come back to this issue in the next section.

4. The Transaspect Phenomenon

The opposite of the second equation in (24) is exactly the constraint (21) [the same as (17)] for the average $\overline{E'}$:

- if $K = -1/2$, as we have just seen, the resulting energy E' is infinite;
- if $K < -1/2$, we obtain imaginary values for $\overline{E'}$.

In one case or the other, the constraint to the average formulae (14) and (15) relates to the existence of critical angles for colliding particles of opposite aspects. But a major surprise still hides!

According to (24), we obtain discontinuity points for an incident *photon hitting a co-electron* if

$$E \geq -\frac{1}{2}E_0 = 4.09 \times 10^{-14}\text{J} \quad \text{or} \quad \lambda \leq 4.85 \times 10^{-2}\text{Å};$$

for an incident *co-photon hitting an electron* it is the opposite:

$$E \leq -\frac{1}{2}E_0 = -4.09 \times 10^{-14}\text{J} \quad \text{or} \quad \lambda \geq -4.85 \times 10^{-2}\text{Å}.$$

The threshold $\lambda_\aleph = 4.85 \times 10^{-2}$Å, which we may call *"aleph wavelength"*, corresponds to an *"aleph energy"* given precisely by $E_\aleph = E_0/2 = 4.09 \times 10^{-14}\text{J} = 2.56 \times 10^5\text{eV}$.

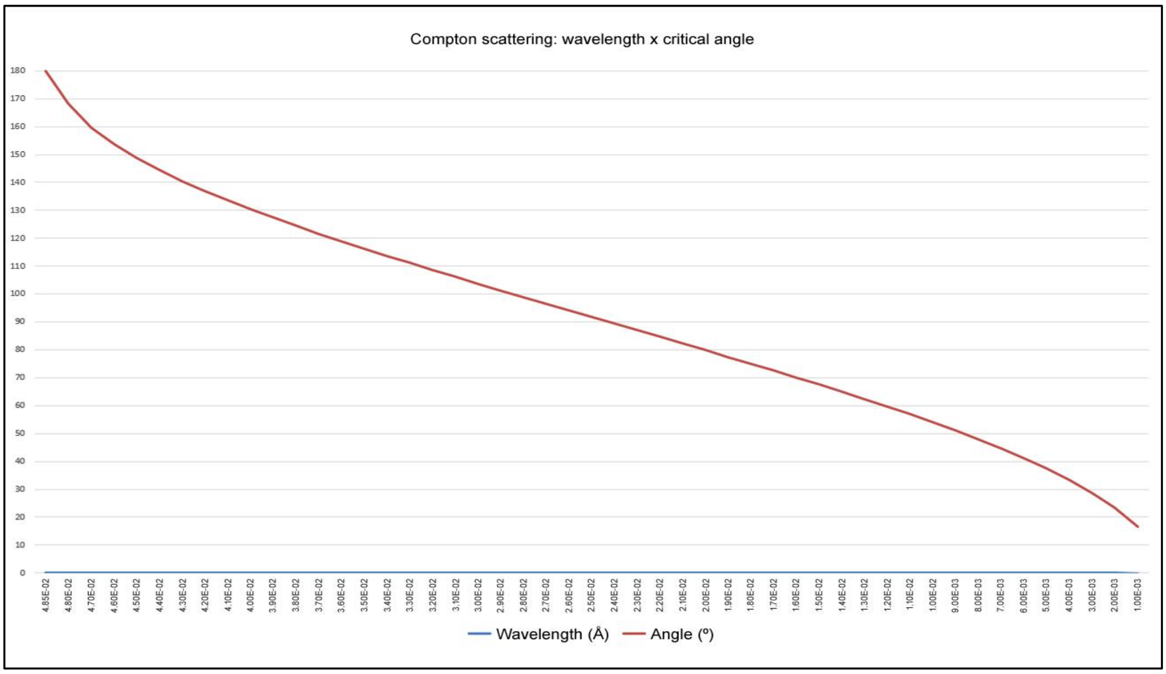
So, we see that critical angles correspond to the highest energetic photons possible (or co-photons, in modulus), from $E_\aleph$ on, in the gamut of gamma rays. In the Figure 7, the curve for the critical angles as a function of the wavelength is represented. Remarkably, for λ ign the gap $(0°, \lambda_\aleph)$, in modulus, the possible values for critical angles covers all the range from 0° to $\pm 180°$; this is, the "critical zone" (which we will discuss ahead) potentially covers the entire space except for the x-axis, in the limits.

An interesting situation is:

$$K = -1, \text{ this is, } E = -E_0 \quad \Rightarrow \quad \lambda = -\lambda_0,$$

which is the inverse of Compton wavelength. In this case, $\varphi_c = \pm 90°$ and *the total energy is null*: $E + E_0 = 0$.

Figure-7. Wavelength x critical angle in Compton scattering for mixed energies (photon ↦ co-electron), beginning with the "aleph wavelength" $\lambda_\aleph = -4.85 \times 10^{-2}$Å



An example: let us study what happens concerning the wavelength

$$\lambda = -4 \times 10^{-2}\text{Å}.$$

We obtain from (15) an imaginary number for the average of kinetic energy: $\overline{E_k} = i \cdot 5.79 \times 10^{-14}$J. Although this is an expected result, the question is: *how can we explain an imaginary number for an average of numbers in which none of them is imaginary?*

Differently, using discrete calculus (making $\varphi = n \cdot 3°$, with $0 \leq n \leq 120$, which do not include the critical angles), all the calculated energies have finite values and the result is the discrete average

$$\overline{[E_k]} = -4.27 \times 10^{-14}\text{J} = -2.66 \times 10^5 \text{ eV};$$

this is a precise, finite number... though *negative!* And, since there are positive values (for instance, $E_k = 9.48 \times 10^3$eV for $\varphi = 18°$ or $E_k = 1.35 \times 10^5$eV for $\varphi = 60°$), there must be negative ones. And there are; for instance, $E_k = -2.66 \times 10^6$eV for $\varphi = 150°$. This is an amazing conclusion!

As a matter or fact, the critical angles form a sort of barrier dividing the domain $[0°, 360°]$ for φ in two regions. Let us see what happens in each one of them. From now on, when needed, we will write the two critical angles in the form

$$0° < \varphi_{c_1} \leq 180° \quad and \quad \varphi_{c_2} = 360° - \varphi_{c_1}, d$$

φ_c representing any of them.

- If φ belongs to the gap $[0°, \varphi_{c_1})$ or $(\varphi_{c_1}, 360°]$, the incident particle and the scattered one are of the **same aspect** (for instance, both co-photons).

In fact, this implies $\cos\varphi > \cos\varphi_c$, or

$$\cos\varphi > \frac{1}{K} + 1 \quad \Rightarrow \quad 1 - \cos\varphi < -\frac{1}{K} \quad \text{or} \quad K(1 - \cos\varphi) > -1,$$

since $K < 1$; but this means, according to (22), that

$$\frac{E}{E'} = 1 + K(1 - \cos\varphi) > 0.$$

• If φ belongs to the gap $(\varphi_{c_1}\ ,\ \varphi_{c_2})$, the incident particle and the scattered one are of **opposite aspects** (for instance, at first a co-photon, then a photon).

We may conclude, in a similar way, that, in this case, $\cos\varphi < \cos\varphi_c$ conduces to

$$\frac{E}{E'} = 1 + K(1 - \cos\varphi) < 0.$$

Let us take a look at this last case, the **transaspect gap**. It includes $\varphi = 180°$; note that this angle corresponds to the maximum ratio $\frac{E}{E'}$ or the ***minimum*** *inverse ratio* $\frac{E'}{E}$:

$$E'_{(\pi)} = \frac{E}{1+2K}.$$

As a matter of fact, if we write $w = \frac{E}{E'} = 1 + K(1 - \cos\varphi)$, then

$$\frac{dw}{d\varphi} = K\sin\varphi = 0 \quad \Rightarrow \quad \begin{cases} \varphi = 0°, \text{ an angle out of the gap; or} \\ \varphi = 180°. \end{cases}$$

Besides, since, for $\varphi = 180°$, $\frac{d^2w}{d\varphi^2} = -K\cos 180° = K < 0$, we may conclude that $\varphi = 180°$ corresponds to a maximum of $w = \frac{E}{E'}$.

It is worth while to study what happens to the massive particle after the interaction, this is $\mathbf{P_4}$, for φ in the transaspect gap $(\varphi_{c_1}\ ,\ \varphi_{c_2})$. Following (22), we obtain its kinetic energy $E_k = E - E'$ from

$$E_k = \frac{K^2(1-\cos\varphi)}{1+K(1-\cos\varphi)}E_0. \tag{25}$$

On the other hand, because of the opposite signs of E and E',

$$E_k = E - E' \quad \Rightarrow \quad |E_k| = |E| + |E'|$$

and therefore, in modulus, the minor value of this kinetic energy corresponds to the minor value of E', this is E'_{π}, or the angle $\varphi = 180°$:

$$E_{k(\pi)} = \frac{2K^2}{1+2K}E_0, \quad \text{which implies} \quad \frac{E_{k(\pi)}}{E_0} = \frac{2K^2}{1+2K} < 0$$

because $2K^2 > 0$ and $K < -1/2$ [or $1 + 2K < 0$]. But this is impossible, for the particle $\mathbf{P_4}$ would have an energy $E_4 = E_0\left(1 + \frac{2K^2}{1+2K}\right)$ in modulus inferior to $|E_0|$... impossible, *unless* $\boldsymbol{P_2}$ *turns into its homologous particle*. Amazingly, this is so; I mean that for its energy $E_4 = E_0 + E_k$ we will have

$$\begin{cases} E_4 \le -E_0 & \text{for } E_0 > 0; \text{ and} \\ E_4 \ge -E_0 & \text{for } E_0 < 0, \end{cases}$$

and I will prove it. To do so, I will use the function $f(K) = K^2 + 2K + 1$. The equation

$$K^2 + 2K + 1 = 0 \quad \text{has a single root:} \quad K = -1.$$

Now, since

$$\begin{cases} \frac{df}{dK} = 2K + 2 = 0 \quad \Rightarrow \quad K = -1; \text{ and} \\ \frac{df^2}{d^2K} = 2 > 0, \end{cases}$$

we conclude that $K = -1$ corresponds to the minimum of the function $f(K)$. We will write then

$$K^2 + 2K + 1 \ge 0 \quad \text{for any } K;$$

but this is equivalent to $K^2 \ge -(1 + 2K)$ or, because $1 + 2K < 0$,

$$\frac{K^2}{1+2K} \le -1 \quad \Rightarrow \quad E_{k(\pi)} = \frac{2K^2}{1+2K}E_0 \begin{cases} \le -2E_0 & \text{for } E_0 > 0; \\ \ge -2E_0 & \text{for } E_0 < 0 \end{cases}$$

Since $E_{4(\pi)} = E_0 + E_{k(\pi)}$, this means precisely that,

$$\begin{cases} E_{4(\pi)} \le -E_0 & \text{for } E_0 > 0; \\ E_{4(\pi)} \ge -E_0 & \text{for } E_0 < 0 \end{cases}$$

Finally, if this conclusion is valid for the lower value of E_k, in modulus, then it is surely valid for any other angle φ in the gap $(\varphi_{c_1}\ ,\ \varphi_{c_2})$.

Remark that the single root $K = -1$ for the equation $f(K) = 0$ reveals a particularly interesting situation:

$$K = -1, \quad \textit{this is} \quad E = -E_0, \quad \Rightarrow \quad \begin{cases} \cos\varphi_c = 0 & \Rightarrow \quad \varphi_c = \pm 90°; \\ E'_{(\pi)} = -E & \\ E_{k(\pi)} = -2E_0 & \Rightarrow \quad E_4 = -E_0. \end{cases}$$

The physical interpretation of the two last lines is what follows: the incident $\mathbf{P_1}$, colliding with $\mathbf{P_2}$, instantly turns into its co-particle and leaves in the place of the hit particle, immobile, this one's homologous co-particle $\widehat{\mathbf{P}}_\mathbf{2}$!

So we see that the critical angles φ_{c_1} and φ_{c_2} perform symmetrical boundaries in relation to the axis defined by the direction of the initial **p** vector; in 3D this corresponds to a conical surface dividing the space in two regions. We will name **critical zone** the region bordered by this conical surface generated by a ray defined by φ_c, its borders excluded. This means that, in the abstract, the critical zone contains all the impulse vectors $\mathbf{p}'$ of scattered massless particles that change aspect under Compton effect.

Then, an amazing final conclusion, for an initial pair *co-photon* ↦ *electron*, arise: it seems that, $\mathbf{p}'$ crossing those critical boundaries, the pair co-photon/electron 'jump' into a pseudotachyonic frame: *the hit electron spontaneously turns into a* ***co-electron****, whilst the incident co-photon turns into a* ***photon****!* A similar conclusion is valid for the time-equivalent collision *photon* ↦ *co-electron*. In both cases, if this is true, it proves that there is indeed no difference in nature between homologous particles.

There is, however, a problem: *there is no strictly conservation of charge* because the massive particle should change sign (unless we deal here with prime-antiparticles, which have the same charge as their homologous particles).

On the other hand, as we have seen, the critical angles lead to corresponding infinite energies (from both the incident and target particles). Let us examine more closely what happens related to the conical frontier of the transaspect gap and beyond. In a first approach, the *infinite critical energy* E'_c for the scattered $\mathbf{P_3}$ related to φ_c matches with a *null critical wavelength* λ'_c:

$$E'_c = \infty \quad \Rightarrow \quad \lambda'_c = \frac{1}{E'_c} = 0.$$

This is compatible with applying (23) in the equation (7) for Compton effect:

$$\Delta\lambda_c = \frac{hc}{E_0}\left[1 - \left(\frac{1}{K} - 1\right)\right] = -\frac{hc}{E_0} \cdot \frac{E_0}{E} = -\lambda \quad \Rightarrow \quad \lambda'_c = 0.$$

Apparently, this result imply that, for the critical angles, the scattered incident particle (say, a co-photon) would not really exist, which is extraordinary and confusing – mainly because the other particle (an electron) would also have an infinite energy. Now, $E_4 = \infty$ is for an electron or a co-electron theoretically moving with the speed of light along the same line of the scattered $\mathbf{P_3}$; in fact,

$$\lim_{E_4\to\infty} \beta^2 = \lim_{E_4\to\infty} 1 + \frac{E_0}{E_4} = 1 \quad \text{and} \quad \lim_{\lambda'\to\infty} \tan\theta = \tan\varphi.$$

There is indeed an increasing velocity of the particle as $\varphi \to \varphi_c$, tending to c. However the speed of light is actually unattainable[5] and, therefore, all this reasoning show that φ_c must be a limit impossible to reach. This is, the discontinuity points φ_c physically correspond to a fundamental discontinuity in the scattering process.

Anyhow, there is a certain logic in presuming that, beyond this frontier, the hit particle becomes tachyonic, existing *as it is* in a **tachyonic frame** S''; and then, its coordinates E''_4 and p''_4, as those of the massless scattered particle, E''_3 and p''_3, directly transform[6] to 'our' S as *imaginary numbers*, which would explain why the average calculus gives imaginary numbers, according to equations (14) to (16). But, in this case, PtR teaches that we may only detect both scattered particles as existing in a correspondent pseudotachyonic frame S^*, supposed, for symmetry reasons, immobile in S (in this case, its *paraframe*[7]). In S^*, the particles do not change aspect; but they do when submitted to pseudotachyonic transformation:

$$\begin{cases} E_3 = -E_3^* & \text{and} \quad E_4 = -E_4^* \\ \mathbf{p}_3 = \mathbf{p}_3^* & \text{and} \quad \mathbf{p}_4 = \mathbf{p}_4^*. \end{cases}$$

This, along with the reversion of charge, $e^* = -e$, really implicates the change of aspect of both particles. Naturally, the conservation of total energy and momentum still holds since it is a basic assumption in obtaining the equations for the Compton effect:

$$\begin{cases} E_3 + E_4 = E_{\mathrm{t}} = E + E_0 \\ \mathbf{p}_3 + \mathbf{p}_4 = \mathbf{p}_{\mathrm{t}} = \mathbf{p} \end{cases}$$

But this means that, even though the initial particles $\mathbf{P}_1$ and $\mathbf{P}_2$ retain their aspect (their positive or negative energy) in the pseudotachyonic frame S^*, within the transaspect zone, it must be $E_{\mathrm{t}}^* = -E_{\mathrm{t}}$. This may be confusing but remark that the same transaspect phenomenon must occur in S^*, in the same conditions; consequently,

$$\begin{cases} E^* = -E4pt \\ E_0^* = -E_0 \end{cases} \quad \Rightarrow \quad E_{\mathrm{t}}^* = E^* + E_0^* = -(E + E_0) \quad \Rightarrow \quad E_{\mathrm{t}}^* = -E_{\mathrm{t}}.$$

This is a quite satisfactory result but seems to implicate a violation in the conservation of **charge**, since this one changes sign. Is this admissible? Perhaps, if we conclude that *conservation principles apply, in fact, to interactions between* ***archeparticles****; and then to its manifestations in different frames of coordinates.* This would be the fundamental reality. Then, if these frames are not of the same kind, opposite signs may arise.

So, for angles φ in the transaspect gap $(\varphi_{c_1}\ ,\ \varphi_{c_2})$, physically, an incident photon ($p > 0$) becomes a co-photon ($p' < 0$) and, conversely, an incident co-photon ($p < 0$) becomes a photon ($p' > 0$). In the first case, $\varphi_s = \varphi - 180°$, in the second, $\varphi_s = 180° - \varphi$; this is,

$$\varphi_s = (-1)^{k_1}(\varphi - 180°). \qquad (26)$$

Since, in general (see Appendix A),

$$\begin{cases} p'_x = p'\cos\varphi_s \\ p'_y = p'\sin\varphi_s, \end{cases}$$

this obliges to rewrite the equations (8) and (9) as

$$\begin{cases} p'_x = -p'\cos\varphi \\ p'_y = (-1)^{k_1+1}p'\sin\varphi \end{cases} \quad \text{and} \quad \begin{cases} p_{4x} = p + p'\cos\varphi \\ p_{4y} = -p'_y = (-1)^{k_1}p'\sin\varphi. \end{cases} \qquad (27)$$

In its turn, the equation for $\tan\theta$ in (10) becomes

$$\tan\theta = \frac{\lambda\sin\varphi}{\lambda\cos\varphi + \lambda'} \quad \text{making, then,} \quad \theta_s = (-1)^{k_1}\theta. \qquad (28)$$

The last condition comes from applying the general rule in (4) to the fact that $\theta_s = \theta$ corresponds to an electron with an incident photon ($k_1 = k_2 = 0$) and $\theta_s = \theta$ to a co-electron with an incident co-photon ($k_1 = k_2 = 1$).

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[5] This is well known. But if not, for instance, the massive particle would turn in a (co-)photon and electric charge would simply vanish.

[6] This means, transforming in agreement with the ordinary Lorentz equations for $\beta^2 > 1$.

[7] In general, the *paraframe* $S^\dagger$ of S^* is the usual bradyonic frame moving with velocity $\hat{v} = c^2/v$. For S^* to be immobile in S, we make $v \to \infty$ in pseudotachyonic transformations; here, yields $E = E^\dagger = -E^*$ and $\mathbf{p} = \mathbf{p}^\dagger = \mathbf{p}^*$ for whatever particle.

The equation (26) correspond to symmetrical possibilities. Now, since φ_{c_1} is the smallest angle in the gap $(\varphi_{c_1}\ ,\ \varphi_{c_2})$, $\varphi_{sc_1} = 180° - \varphi_{c_1}$ is the largest possible. We will name **transaspect zone** the region of the three-dimensional space bordered by the conical surface generated by a ray making an angle φ_{sc} with the x-axis. Note that this is not a strange region of the space itself; it just contains all the scattered (co-)photons that change aspect under Compton effect and here lies its peculiarity, just concerning the phenomenon.

In our example above, the wavelength $\lambda = -4 \times 10^{-2}$Å, one gets for the critical angle:

$\varphi_c = \pm 130.44°$.

The conical zone corresponding to this situation is represented on Figure 8, as well as two possible results, each on one side or the other of the frontier.

1. The outcome for $\varphi = 60°$, this is $\varphi_s = -60°$ and $\theta_s = -102.80°$, is an identical pair *co-photon/electron*, with $\lambda' = -2.78 \times 10^{-2}$Å,

$E' = 7.13 \times 10^{-14}\mathrm{J}$;

$$p' = -2.38 \times 10^{-22} \begin{cases} p'_x = -1.19 \times 10^{-22} \\ p'_y = 2.06 \times 10^{-22} \end{cases} \text{kg m/s;}$$

and

$E_4 = 1.03 \times 10^{-13}\mathrm{J}$;

$$p_4 = 2.11 \times 10^{-22} \begin{cases} p_{4x} = -4.68 \times 10^{-23} \\ p_{4y} = -2.06 \times 10^{-22} \end{cases} \text{kg m/s}$$

for the electron.

2. The outcome for $\varphi = 150°$, this is $\varphi_s = 30°$ and $\theta_s = 26.61°$, is an opposite pair *photon/co-electron,* with $\lambda' = 5.28 \times 10^{-3}$Å,

$E' = 3.77 \times 10^{-13}\mathrm{J}$;

$$p' = 1.26 \times 10^{-22} \begin{cases} p'_x = 1.09 \times 10^{-21} \\ p'_y = 6.28 \times 10^{-22} \end{cases} \text{kg m/s;}$$

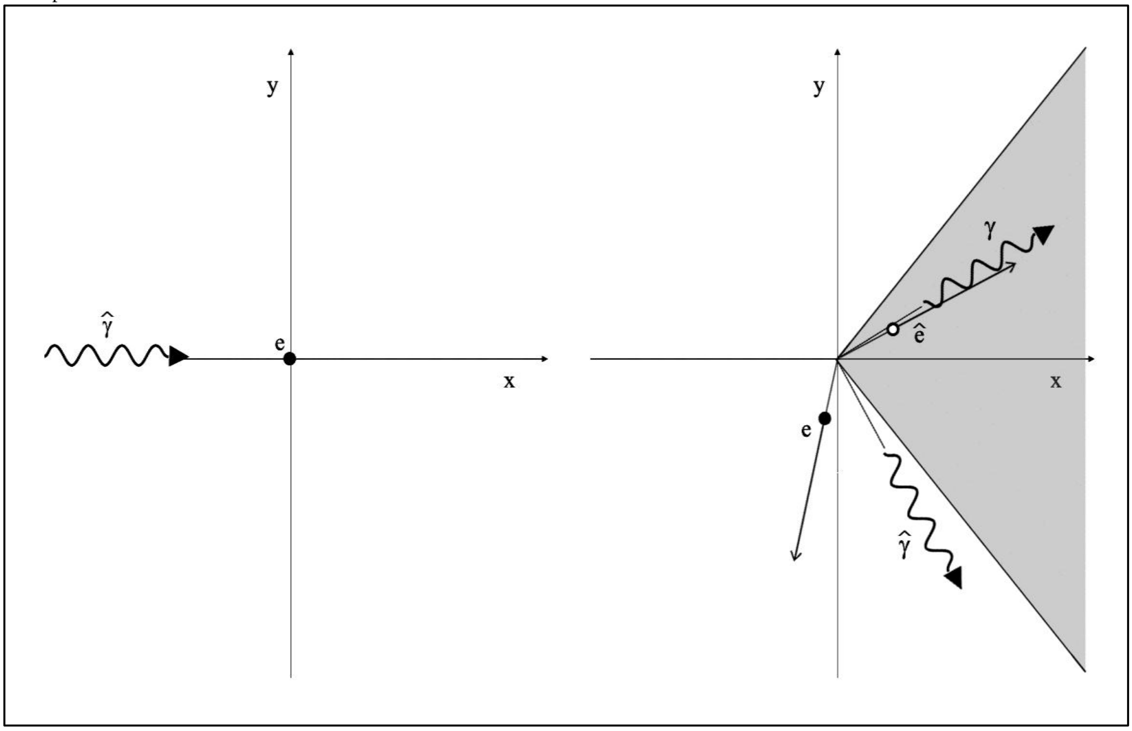
and

$E_4 = -3.44 \times 10^{-13}\mathrm{J}$;

$$p_4 = -1.12 \times 10^{-21} \begin{cases} p_{4x} = -1.25 \times 10^{-21} \\ p_{4y} = -6.28 \times 10^{-22} \end{cases} \text{kg m/s}$$

for the co-electron.

Figure-8. Two distinct results for a **co-photon** colliding with an **electron**: 1) an identical pair co-photon/electron; and 2) a pair photon/co-electron in the transaspect zone



For another angle in the transaspect zone (a particular one), $\varphi = 180°$, $\varphi_s = 0°$ and $\theta_s = 0°$, we obtain for the scattered co-photon (now photon):

$\lambda' = 4.85 \times 10^{-2}$Å $\Leftrightarrow$ $E' = 2.33 \times 10^{-13}$J.

and for the ex-electron, now a co-electron:

$E_k = -2.83 \times 10^{-13}$J and $E_4 = -2.01 \times 10^{-13}$J.

These last results correspond simultaneously to the lowest energy of the photon and (in modulus) of the co-electron generated by the transaspect process. This may seem strange but it is quite logical; one must remember that, for mixed particles involved in Compton scattering, *in modulus* the gain of energy of one of them implicates a gain of energy on the other, because their energies have opposite signs.

It is noteworthy that the discrete average for kinetic energy within the transaspect zone is, in modulus, significantly higher ($\approx 3.41\times$) than outside of it:

$$\overline{[E_k^-]} = -7.01\times10^{-13}\text{J},\quad \text{for}\quad \overline{[E_k^+]} = 2.05\times10^{-13}\text{J}.$$

This obviously relates to the fact that the total discrete average is negative.

Finally, let us investigate the relationship between the negative K factor and the critical angles φ_c. From (23), we may write

$$\cos\varphi_c = 1 - \frac{1}{|K|},$$

and this means that:

- $\cos\varphi_c$ grows – and φ_{c_1} decreases – with growing $|K|$;
- $\cos\varphi_c$ decreases – and φ_{c_1} grows – with decreasing $|K|$;

or: the transaspect gap *narrows* as $K = E/E_0$ varies from $-1/2$ to $-\infty$.

In particular, keeping in mind that $\varphi_{sc} = (-1)^{k_1}(\varphi_c - 180°)$:

- $\lim_{K\to\infty}\cos\varphi_c = 1$ or $\lim_{K\to\infty}\varphi_c = 0°$;

in this theoretical limit, corresponding to $E_0 \to 0$ for constant E, the scattered massless $\mathbf{P}_3$ would move in the opposite sense of the incident $\mathbf{P}_1$.

- $K = -1$ [this is, $E = -E_0$, as we have seen]: $\cos\varphi_c = 0$ or $\varphi_c = \pm 90°$;

the transaspect zone is now the frontal semi-space (in the direction of the x-axis).

- $K = -\frac{1}{2}$ [this is, $E = -2E_0$]: $\cos\varphi_c = -1$ or $\varphi_c = \pm180°$;

the transaspect zone resumes to the positive part of the x-axis.

5. Conclusion

We have generalized the study of Compton effect to co-particles, with the comprehension that there is no difference in nature between particles and homologous co-particles: they are just two aspects of the same *archeparticle*. This brought some expected answers but also some surprises, concerning mixed energies, such as:

1. The increasing energy (in modulus) of the incident photon or co-photon when hitting respectively a co-electron or an electron.
2. In certain conditions, the *transaspect* phenomenon, this is, the transmutation of both colliding particles into their homologous ones, within a conical zone around the axis defined by the movement of the incident particle.

These assertions should possibly be experimentally verified, for instance by making photons to collide with co-electrons (if one can provide them).

Finally, this paper is also intended to constitute a subsidy for an electrostatic or gravitational field theory based, on the one hand, on De Broglie's "periodic process" and, on the other hand, on the generalized Compton effect. The foundations of this theory have already been presented in other articles.

Appendix A. On the Compton Effect Basics

We obtain here several equations presented in section 2.

1) We start with those concerning the **scattering angles**. Let φ and θ be the angles the vectors $\mathbf{p_3}$ and $\mathbf{p_4}$ form with the vector $\mathbf{p_1}$ [see figure 2]. If the incoming particle is a *photon* (positive energy), the scattering angles for both particles – relatively to the *xx* axis positive sense – are

$$\begin{cases}\varphi_s = \varphi & \text{for the photon; and}\\ \theta_s = k_2\cdot180° + \theta & \text{for the electron/co}-\text{electron}\quad (k_2 = 0\ /\ k_2 = 1).\end{cases}$$

If the incoming particle is a *co-electron* (with negative energy), the scattering angles for both particles are

$$\begin{cases}\varphi_s = -\varphi & \text{for the co}-\text{photon; and}\\ \theta_s = (1-k_2)\cdot180° - \theta & \text{for the electron/co}-\text{electron.}\end{cases}$$

Therefore, we may unite these two equation system in a single one:

$$\begin{cases}\varphi_s = (-1)^{k_1}\cdot\varphi\\ \theta_s = k_1\cdot180° + (-1)^{k_1}(k_2\cdot180° + \theta),\end{cases}$$

where $k_i = 0$ for particles and $k_i = 1$ for co-particles.

2) In obedience to the conservation of the linear momentum, it must be

$$\mathbf{p_4} = \mathbf{p_1} - \mathbf{p_3}$$

and so (making $E_1 = E$ and $E_3 = E'$ for the non-massive particle and also $E_2 = E_0$ for the massive one)

$$p_4^2 = p_1^2 + p_3^2 - 2\mathbf{p_1}\cdot\mathbf{p_3} = \frac{1}{c^2}(E^2 + E'^2 - 2EE'\cos\varphi);\,(a)$$

on the other hand, the conservation of energy law implies that

$$E + E_0 = E' \pm\sqrt{E_0^2 + c^2p_4^2},$$

which means

$$p_4^2 = \frac{1}{c^2}[E^2 + E'^2 - 2EE' + 2(E - E')E_0];\,(b)$$

finally, equalizing the second terms of equations (a) and (b), it results

$$(E - E')E_0 = EE'(1 - \cos\varphi),$$

or

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{E_0}(1 - \cos\varphi),$$

which is the mathematical expression for the Compton effect. Notice that this expression remains valid for *co-particles* because the signs of the variables energy or linear momentum have no relevance in its deduction.

3) Concerning the components of the momentum p', we obtain for incident *photons*, with $\varphi_s = \varphi$:

$$\begin{cases} p'_x = p'\cos\varphi \\ p'_y = p'\sin\varphi, \end{cases} \quad \text{where} \quad p' = \frac{h}{\lambda'}.$$

But, for incident *co-photons*, where p and p' are negative, the equivalent situation to this one obliges to consider $-\varphi$ in order to obtain an identical $\varphi_s = -\varphi$; therefore,

$$\begin{cases} p'_x = p'\cos\varphi_s = p'\cos\varphi \\ p'_y = p'\sin\varphi_s = -p'\sin\varphi, \end{cases}$$

and we may describe both situations by

$$\begin{cases} p'_x = p'\cos\varphi \\ p'_y = (-1)^{k_1}\, p'\sin\varphi. \end{cases}$$

This may seem strange. But one may easily understand the negative sign in p'_y as follows: take a positive $\varphi < 90°$; since p is negative (opposite to the xx axis), for a resulting negative $p' = E'/c$, p'_x is also negative but p'_y must be positive; it would result negative for $p'\sin\varphi$.

4) For the kinetic energy:

$$E_k = E - E' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\,\frac{\lambda' - \lambda}{\lambda\,\lambda'} = hc\,\frac{\Delta\lambda}{\lambda\,\lambda'}.$$

5) For the velocity of the hit particle, where $E_4 = E_0 + E_k$:

$$E_4 = \frac{E_0}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad 1 - \beta^2 = \frac{E_0^2}{E_4^2} \quad \Rightarrow \quad \beta^2 = 1 - \frac{E_0^2}{(E_0 + E_k)^2} = \frac{E_4^2 - E_0^2}{E_4^2}.$$

6) For the total momentum p_4:

$$p_4 = \frac{E_4}{\hat{v}} = \frac{E_4/c}{\hat{\beta}} = \beta\,\frac{E_4}{c} \quad \Rightarrow \quad p_4 = \frac{\sqrt{E_4^2 - E_0^2}}{|E_4^2|} \cdot \frac{E_4}{c} = \frac{(-1)^{k_2}}{c}\sqrt{E_4^2 - E_0^2},$$

because E_4 is negative for a co-electron. Remark that $E_4^2 - E_0^2 = (E_k + E_0)^2 - E_0^2 = E_k^2 + 2EE_0$ and, so,

$$p_4 = \frac{(-1)^{k_2}}{c}\sqrt{E_k(E_k + E_0)}.$$

Alternatively, making $\hat{\beta} = \hat{v}/c = 1/\beta$, according to equation (2):

$$p_4 = \frac{E_4}{\hat{v}} = \frac{E_4}{c\hat{\beta}} = E_4\beta/c.$$

7) For the angle θ, we need to understand that this angle is measured with respect to a coordinate basis [2] where the $xx^{[2]}$ axis is pointed in the direction of the vector **p**. This means that the change of basis from 'our' basis [0] to [2] provides:

$$\begin{cases} x^{[2]} = (-1)^{k_1}\, x \\ y^{[2]} = y \end{cases} \quad \textit{and also} \quad \begin{cases} p_{4x}^{[2]} = (-1)^{k_1}\, p_{4x} \\ p_{4y}^{[2]} = p_{4y}. \end{cases}$$

Therefore, in the condition that $p_{4x} \neq 0$, and according to equations (9):

$$\tan\theta = \frac{p_{4y}^{[2]}}{p_{4x}^{[2]}} = -\frac{p'\sin\varphi}{p - p'\cos\varphi} = \frac{\sin\varphi}{\cos\varphi - \lambda'/\lambda},$$

this is,

$$\tan\theta = \frac{\lambda\sin\varphi}{\lambda\cos\varphi - \lambda'}.$$

Alternatively,

$$\frac{E}{E'} = \frac{\lambda'}{\lambda} = \frac{\lambda + \Delta\lambda}{\lambda} = 1 + \frac{\Delta\lambda}{\lambda} \quad \Rightarrow \quad \frac{E}{E'} = 1 + \frac{h}{m_0 c\lambda}(1 - \cos\varphi); \qquad (c)$$

on the other hand, for $\sin\varphi \neq 0$,

$$\cot\theta = \frac{p_{4x}^{[2]}}{p_{4y}^{[2]}} = -\frac{p - p'\cos\varphi}{p'\sin\varphi} = -\left(\frac{E}{E'} - \cos\varphi\right)\frac{1}{\sin\varphi},$$

and so, applying the equation (c),

$$\begin{aligned} \cot\theta &= -\left[1 + \frac{h}{m_0 c\lambda}(1 - \cos\varphi) - \cos\varphi\right]\frac{1}{\sin\varphi} 4pt \\ &= -\left(1 + \frac{h}{m_0 c\lambda}\right)\frac{1 - \cos\varphi}{\sin\varphi}; \end{aligned}$$

but

$$\frac{1 - \cos\varphi}{\sin\varphi} = \frac{2\sin^2\frac{\varphi}{2}}{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}} = \frac{\sin\frac{\varphi}{2}}{\cos\frac{\varphi}{2}} = \tan\frac{\varphi}{2}$$

and we finally obtain Debye's formula (adapted with the - sign):

$$\cot\theta = -\left(1 + \frac{h}{m_0 c\lambda}\right)\tan\frac{\varphi}{2}.$$

Appendix b. On the Average Energy

An algebraic expression for the integral **I** in the average energy $\overline{E'}$ equation,

$$\overline{E'}\Big|_0^{2\pi} = \frac{E}{2\pi}\int_0^{2\pi}\frac{d\varphi}{1+E/E_0(1-cos\varphi)},$$

may be deduced by the method of changing variable:

$$t = \tan\frac{\varphi}{2} \quad \Rightarrow \quad \cos\varphi = \frac{1-t^2}{1+t^2} \quad \text{and} \quad d\varphi = \frac{2dt}{1+t^2}.$$

As a matter of fact,

$$\mathbf{I} = \int \frac{\frac{2dt}{1+t^2}}{1+G\left(1-\frac{1-t^2}{1+t^2}\right)} = \frac{2}{1+2G}\int\frac{dt}{\left(\frac{1}{1+2G}+t^2\right)} = 2M\int\frac{dt}{M+t^2}$$

making $M = \frac{1}{1+2G} = \frac{E_0}{E_0+2E}$. But this means that

$$\mathbf{I} = 2M\left(\frac{1}{\sqrt{M}}\arctan\frac{t}{\sqrt{M}}\right)$$

or, finally,

$$\overline{E'}\Big|_0^{2\pi} = \frac{E}{\pi}\sqrt{M}\arctan 0.$$

Here, there are two alternatives: the first $[\arctan 0 = 0]$ is a misleading one, for it corresponds to $\overline{E'} = 0$, which seems quite strange and is false. The second one $[\arctan 0 = \pi]$ proves to be the correct alternative; it leads to

$$\overline{E'} = E\sqrt{\frac{E_0}{E_0+2E}}\ .$$

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