Sumerianz Journal of Scientific Research, 2020, Vol. 3, No. 10, pp. 122-128
ISSN(e): 2617-6955, ISSN(p): 2617-765X
Website: https://www.sumerianz.com
DOI: https://doi.org/10.47752/sjsr.310.122.128

Original Article Open Access

Effective Interfacial Tension Effect on Kelvin-Helmholtz Instability

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Article History
Received: September 10, 2020
Revised: October 15, 2020
Accepted: October 18, 2020
Published: October 20, 2020

Abstract
The instability of the plane interface between two uniform, superposed and streaming Walters′ B′ viscoelastic fluids through porous medium in the presence of effective interfacial tension is considered. The case of two uniform streaming fluids separated by a horizontal boundary is studied. It is observed, for the special case where the effective interfacial tension is ignored, that the system is stable or unstable for the potentially stable configuration which is in contrast to the case of Rivlin-Ericksen viscoelastic fluid or Newtonian fluid where the system is always stable for the potentially stable configuration. Moreover, if the perturbations in the direction of streaming are ignored, then the perturbations transverse to the direction of streaming are found to be unaffected by the presence of streaming, whereas for perturbations in all other directions there exists instability for a certain wave number range. 'Effective interfacial tension' is able to suppress this Kelvin-Helmholtz instability for small wavelength perturbations, the medium porosity reduces the stability range given in terms of a difference in streaming velocities.
Keywords: Kelvin-helmholtz instability; Walters′ B′ viscoelastic fluid; Effective interfacial tension; Porous medium.

1. Introduction
When two superposed fluids flow over the other with a relative horizontal velocity, the instability of the plane interface between the two fluids is known as the "Kelvin-Helmholtz instability". The discontinuity arising at the plane interface between two superposed streaming fluids is of prime importance in various astrophysical and laboratory situations. The Kelvin-Helmholtz instability arise in situations such as air blowing over mercury, highly ionized hot plasma surrounded by a slightly colder gas, or a meteor entering the earth's atmosphere. The instability of the plane interface between two superposed semi-infinite inviscid fluids flowing with different velocities has been considered by Helmholtz [1] and Kelvin [2] and a review of this Kelvin-Helmholtz instability, under varying assumptions of hydrodynamics and hydromagnetics, was given by Chandrasekhar [3]. These problems of instabilities in hydrodynamic and hydromagnetic configuration continue to attract the attention of researchers due to their importance in actual physical situations. Some experimental observation of the Kelvin-Helmholtz instability has been given by Francis [4]. The stability problem of non-conducting, streaming gas flowing over an incompressible conducting fluid has been studied by Gerwin [5]. The effect of rotation and a general oblique magnetic field on Kelvin-Helmholtz instability was studied by Sharma and Srivastava [6]. Mehta and Bhatia [7], studied the Kelvin-Helmholtz instability of two viscous, superposed plasmas in the presence of finite Larmor radius (FLR) effects and showed that both viscosity and FLR effects suppressed the instability. An excellent reappraisal of the Kelvin-Helmholtz problem was made by Benjamin and Bridges [8], who gave the Hamiltonian formulation of the classic Kelvin-Helmholtz problem in hydrodynamics. Allah [9], investigated the effects of magnetic field, heat and mass transfer on the Kelvin-Helmholtz instability of superposed fluids. The medium was assumed to be non-porous and fluids Newtonian in the above studies.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. The problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation has been studied by [10] and found that rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. Sharma [11], studied the problem of thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation. There are many viscoelastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of viscoelastic fluids is the Walters B′ viscoelastic fluid Walters [12] having relevance and importance in geophysical fluid dynamics, chemical technology, and petroleum industry. Walters [13], reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters B′ viscoelastic fluid. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics engineering equipments, contact lens, etc. Walters B′ viscoelastic fluids form the basis for the manufacture of many important and useful products. The flow of an unsteady viscoelastic (Walters B′ liquid) conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different

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angular velocities in the presence of a uniform axial magnetic field has been studied by Chakraborty and Sengupta [14]. Sharma and Kumar [15], studied the stability of the plane interface separating two viscoelastic (Walters B′) superposed fluids of uniform densities. In another study, [16] studied Rayleigh-Taylor instability of superposed conducting Walters B′ viscoelastic fluids in hydromagnetics. Kumar [17], considered the thermal instability of a layer of a Walters B′ viscoelastic fluid acted on by a uniform rotation and found that for stationary convection, rotation has a stabilizing effect. Kumar, *et al.* [18], considered the stability of the plane interface separating two Walters B′ viscoelastic superposed fluids of uniform densities in the presence of suspended particles. The medium was non-porous in all the above studies.

Recently, interest in viscoelastic flows through porous media has grown considerably, due largely to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agricultural engineering to study the underground water resources., seepage of water in river beds. A great number of applications in geophysics may be found in the book by Phillips [19]. The gross flow of fluid slowly percolating through rock pores is described by Darcy's law. As a result, the usual viscous term in the equations of motion for a Walters B′ viscoelastic fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the fluid, k_1 is the medium permeability, and $\vec{q}$ is the Darcian (filter) velocity of the fluid. Generally, it is accepted that comets consist of a dusty "snowball" of a mixture of frozen gases, which in the process of their journey, alternate between solid to gas phases. The physical properties of comets, meteorites, and interplanetary dust strongly suggest the importance of porosity in astrophysical contexts [20]. Kumar and Singh [21], have studied the instability of the plane interface between two Walters B′ viscoelastic superposed fluids permeated with suspended particles and uniform rotation in porous medium. In another study, Kumar and Kumar [22] have studied the thermosolutal convection in Walters B′ heterogeneous viscoelastic fluid through Brinkman porous medium.

A theoretical and experimental investigation of the instability of slow, immiscible, viscous liquid-liquid displacements in porous media was presented by Chouke, *et al.* [23]. In flows through porous media, the front is not sharp (as in ordinary fluid dynamics) but is dispersed and broad; [23] assumed a macroscopic interface and an 'effective interfacial tension'. The instability of the plane interface between two uniform superposed and streaming fluids through a porous medium was investigated by Sharma and Spanos [24].

Keeping in mind the different importance stated above of viscoelastic fluid and porous media, the problem of instability of streaming Walters B′ viscoelastic fluids in a porous medium including an effective interfacial tension effects is considered here. The problem has importance in chemical technology, oil recovery, and modern industries.

2. Formulation of the Problem and Perturbation Equations

The initial stationary state, whose stability we wish to examine is that of an incompressible Walters B′ viscoelastic fluid in porous medium in which there is a horizontal streaming $\vec{U}(U(z),0,0)$. The stability of an initial state is determined by supposing that the system is slightly disturbed; its evolution (or lack thereof) is then followed.

In flows through porous media, there are no sharp fronts and so no actual interfacial tensions at some prescribed levels z_s, as in ordinary fluid dynamics. However, there is a macroscopic interface (broad front), if viewed from a large distance, and by analogy with Laplace's formula, at each point of the macroscopic interface

$$(p_1 - p_2)_{z=z_s} = -T_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \qquad (1)$$

where R_1, R_2 are the signed radii of curvatures of the macroscopic interface and T_s is the 'effective interfacial tension'. This is the first approximation to the problem in porous medium and this theory was used and elucidated by Chouke, *et al.* [23]. The "effective interfacial tension" is a measurement of the cohesive (excess) energy present at an interface arising from the imbalance of forces between molecules at an interface. When two different phases are in contact with each other, the molecules at the interface experience an imbalance of forces. This will lead to an accumulation of free energy at the interface. The excess energy is called surface energy and can be quantified as a measurement of energy/area, i.e. the energy required to increase the surface area of the interface by a unit amount. It is also possible to describe this situation as a force/length measurement. This force tends to minimize the area of the surface, thus explaining why for example liquid drops and air bubbles are round. This excess energy exists at any interface. The "effective interfacial tension" depends upon the parameters like, relative permeability-saturation-fluid process functional relationships in two-fluid phase porous media system. The difficulty in determining the "effective interfacial tension" limits the prediction of the wavelength of fingering of immiscible fluids in porous media. A method to estimate the "effective interfacial tension" using fractal concepts was presented by Chang, *et al.* [25] and modified by Smith and Zhang [26] taking the macroscopic interface length instead of the system width.

Let $\vec{q}(u,v,w)$, δp and $\delta\rho$ denote respectively the perturbations in velocity $\vec{U}(U(z),0,0)$, pressure p and density ρ. Let the discontinuity in density occur at $= z_s$, which after perturbation becomes

$$z_s + \delta z_s(x,y,t). \qquad (2)$$

Therefore, on account of (1), the discontinuity in the normal stresses required for equilibrium is

$$T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\,. \tag{3}$$

Hence, the linearized perturbation equations of motion and continuity of Walters B′ viscoelastic fluid in porous medium are Sharma and Spanos [24]; Walters [13]

$$\frac{\rho}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{U}.\nabla)\vec{q}+\frac{1}{\varepsilon}(\vec{q}.\nabla)U(z)\hat{\imath}\right]=-\nabla\delta p+\vec{g}\delta\rho-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\vec{q}+$$
$$+\vec{n}_s\left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\right]\delta(z-z_s)\,, \tag{4}$$

$$\nabla.\vec{q}=0\,, \tag{5}$$

where $\vec{n}_s$ denote the normal to the macroscopic interface. ν,ν',ε and k_1 stand for kinematic viscosity, kinematic viscoelasticity, medium porosity and medium permeability respectively. Here $\vec{g}(=0,0,-g)$ denotes the acceleration due to gravity and $\delta(z-z_s)$ stands for Dirac delta function.

Since the density of a particle moving with the fluid remains unchanged, we have

$$\left[\varepsilon\frac{\partial}{\partial t}+\vec{U}.\nabla\right]\delta\rho=-w\frac{d\rho}{dz}\,. \tag{6}$$

In equation (4), δz_s can be expressed in terms of the normal component of the velocity w_s at z_s because

$$\varepsilon\frac{d}{dt}\delta z_s=w(z_s)=w_s$$

i.e.

$$\left(\varepsilon\frac{\partial}{\partial t}+U_s\frac{\partial}{\partial x}\right)\delta z_s=w_s\,, \tag{7}$$

where the subscript "s" indicates the value of the quantity at $z=z_s$.

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is of the form

$$exp\left[i\left(k_x x+k_y y+nt\right)\right]\,, \tag{8}$$

where n is the rate at which the system departs from the equilibrium, $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave number and k_x, k_y are horizontal wave numbers.

Substituting for $\delta\rho,\delta z_s$ in equation (4) with the help of equations (5), (6), (7) and expression (8), we obtain

$$\left[\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right]\vec{q}+\frac{\rho}{\varepsilon^2}w(DU)\hat{\imath}=-\nabla\delta p+i\vec{g}\frac{(D\rho)}{(\varepsilon n+k_xU)}w+$$
$$+ik^2T_s\left(\frac{w}{\varepsilon n+k_xU}\right)\delta(z-z_s)\,, \tag{9}$$

where $\hat{\imath}$ is the unit vector in the x-direction and $D=\frac{d}{dz}$.

Writing the three component equations of (9) and eliminating u, v and δp with the help of equation (5), we get

$$D\left[\left\{\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right\}Dw-\frac{ik_x\rho}{\varepsilon^2}(DU)w\right]$$
$$-k^2\left[\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right]w=igk^2\left[(D\rho)-\frac{k^2}{g}T_s\delta(z-z_s)\right]\frac{w}{\varepsilon n+k_xU}\,. \tag{10}$$

3. Two Uniform Streaming Walters′ Fluids Separated by a Horizontal Boundary

Here we consider the case when two superposed streaming Walters′ fluids of uniform densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 and uniform viscoelasticites μ_1' and μ_2' are separated by a horizontal boundary at $z=0$. The subscripts 1 and 2 distinguish the lower and the upper fluids respectively. The density ρ_2 of the upper fluid is taken to be less than the density ρ_1 of the lower fluid so that, in the absence of streaming, the configuration is stable and the porous medium throughout is assumed to be isotropic and homogeneous. Let the two fluids be streaming with constant velocities U_1 and U_2. Then in each of the two regions of constant ρ,μ,μ' and U, equation (10) reduces to

$$(D^2-k^2)w=0. \tag{11}$$

The boundary conditions to be satisfied are as follows:

(i) Since U is discontinuous at $z=z_s$, the uniqueness of normal displacement of any point on the interface according to (7) implies that

$$\frac{w}{\varepsilon n+k_xU} \tag{12}$$

must be continuous at an interface.

(ii) Integrating equation (10) between $z_s-\eta$ and $z_s+\eta$ and taking the limit $\eta=0$, we obtain, in view of (12), the jump condition

$$\Delta_s\left[\left\{\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right\}Dw-\frac{ik_x\rho}{\varepsilon^2}(DU)w\right]$$
$$=igk^2\left[\Delta_0(\rho)-\frac{k^2T_s}{g}\right]\left(\frac{w}{\varepsilon n+k_xU}\right)_0,\qquad (for\ z=z_s) \tag{13}$$

while the equation valid everywhere else $(z\neq z_s)$ is

$$D\left[\left\{\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right\}Dw-\frac{ik_x\rho}{\varepsilon^2}(DU)w\right]-k^2\left[\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(\nu-in\nu')\right]w$$
$$=igk^2\left[D\rho-\frac{k^2T_s}{g}\delta(z-z_s)\right]\left(\frac{w}{\varepsilon n+k_xU}\right). \tag{14}$$

Here $\Delta_s(f)=f(z_s+0)-f(z_s-0)$ is the jump which a quantity f experiences at the interface $z=z_s$; and the subscript s distinguishes the value a quantity, known to be continuous at an interface, takes at the interface.

The general solution of equation (11) is a linear combination of the integrals e^{+kz} and e^{-kz}. Because $\left(\frac{w}{\varepsilon n+k_xU}\right)$ must be continuous on the surface $z=z_s$ and w cannot increase exponentially on either side of the interface, the solutions appropriate for the two regions are

$$w_1=A(\varepsilon n+k_xU_1)e^{+kz}, \qquad (z<0) \tag{15}$$
$$w_2=A(\varepsilon n+k_xU_2)e^{-kz}, \qquad (z>0) \tag{16}$$

Applying the boundary condition (13) to the solutions (15) and (16), yields the dispersion relation

$$\left[1-\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right]n^2+\left[\frac{2k_x}{\varepsilon}(\alpha_1U_1+\alpha_2U_2)-\frac{k_x}{k_1}(\alpha_1\nu_1'U_1+\alpha_2\nu_2'U_2)-\frac{i\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]n$$
$$+\left[\frac{k_x^2}{\varepsilon^2}(\alpha_1U_1^2+\alpha_2U_2^2)-\frac{ik_x}{k_1}(\alpha_1\nu_1U_1+\alpha_2\nu_2U_2)-gk\left\{(\alpha_1-\alpha_2)+\frac{k^2T_s}{g(\rho_1+\rho_2)}\right\}\right]=0, \tag{17}$$

where

$$\alpha_{1,2}=\frac{\rho_{1,2}}{\rho_1+\rho_2},\qquad \nu_{1,2}=\frac{\mu_{1,2}}{\rho_{1,2}},\qquad \nu_{1,2}'=\frac{\mu_{1,2}'}{\rho_{1,2}}$$

and

$$\nu_1\left(=\frac{\mu_1}{\rho_1}\right),\ \nu_1'\left(=\frac{\mu_1'}{\rho_1}\right),\qquad \nu_2\left(=\frac{\mu_2}{\rho_2}\right),\nu_2'\left(=\frac{\mu_2'}{\rho_2}\right)$$

are kinematic viscosities and kinematic viscoelasticities of lower and upper fluids, respectively.

Equation (17) is a quadratic equation in $'in'$ and yields

$$in=\frac{1}{2}\left(1-\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right)^{-1}\left[-\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)-\frac{2ik_x}{\varepsilon}(\alpha_1U_1+\alpha_2U_2)+\frac{ik_x}{k_1}(\alpha_1\nu_1'U_1+\alpha_2\nu_2'U_2)\right]$$
$$\pm\frac{1}{2}\left(1-\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right)^{-1}\left\{\left[\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]^2-\frac{4ik_x\alpha_1\alpha_2}{k_1}(\nu_1-\nu_2)(U_1-U_2)-\right.$$
$$-\frac{4k_x^2\alpha_1\alpha_2}{\varepsilon k_1}(\nu_2'U_1-\nu_1'U_2)(U_1-U_2)+\frac{2i\varepsilon k_x}{k_1^2}[(\alpha_1^2\nu_1\nu_1'U_1+\alpha_2^2\nu_2\nu_2'U_2)+\alpha_1\alpha_2$$
$$(\nu_1\nu_2'U_1+\nu_1'\nu_2U_2)+\alpha_1\alpha_2(U_1-U_2)(\nu_1\nu_2'-\nu_1'\nu_2)]-\left[\frac{k_x}{k_1}(\alpha_1\nu_1'U_1+\alpha_2\nu_2'U_2)\right]^2+$$
$$\left.+\frac{4\alpha_1\alpha_2k_x^2}{\varepsilon^2}(U_1-U_2)^2-4gk\left[(\alpha_1-\alpha_2)+\frac{k^2T_s}{g(\rho_1+\rho_2)}\right]\left[1+\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right]\right\}^{1/2}. \tag{18}$$

4. Discussion

4.1. Case I: In the Absence of 'Effective Interfacial Tension'

In the absence of 'effective interfacial tension', some interesting cases are now considered:

(a) when $k_x=0$, equation (18) yields

$$in=\frac{1}{2\left\{1-\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right\}}\left[-\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\pm\left\{\left[\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]^2+\right.\right.$$
$$\left.\left.+4gk(\alpha_2-\alpha_1)\left[1-\frac{\varepsilon}{k_1}(\alpha_1\nu_1'+\alpha_2\nu_2')\right]\right\}^{1/2}\right]. \tag{19}$$

Here we assume that kinematic viscosities ν_1,ν_2 and kinematic viscoelasticities ν_1',ν_2' of the two fluids to be equal i.e. $\nu_1=\nu_2=\nu$, $\nu_1'=\nu_2'=\nu'$. However, any of the essential features of the problem are not obscured by this simplifying assumption. Equation (19) then becomes

$$in=\frac{1}{2\left(1-\frac{\varepsilon\nu'}{k_1}\right)}\left[-\frac{\varepsilon\nu}{k_1}\pm\left[\left(\frac{\varepsilon\nu}{k_1}\right)^2+4gk(\alpha_2-\alpha_1)\left\{1-\frac{\varepsilon\nu'}{k_1}\right\}\right]^{1/2}\right]. \tag{20}$$

(i) Unstable Case: For the potentially unstable configuration ($\rho_2>\rho_1$), it is evident from equation (20) that one of the values of $'in'$ is positive implying that the system is unstable if

$$\nu'<\frac{k_1}{\varepsilon}, \tag{21}$$

whereas both the values of $'in'$ are either real, positive or complex conjugates with positive real parts, again implying instability of the system if

$$\nu' > \frac{k_1}{\varepsilon} \quad . \tag{22}$$

(ii) Stable Case: For the potentially stable configuration $(\rho_2 < \rho_1)$, equation (20) yields that the system is unstable or stable according as

$$\nu' > \frac{k_1}{\varepsilon} \quad or \quad \nu' < \frac{k_1}{\varepsilon} \quad . \tag{23}$$

We, therefore, conclude that for the special case when perturbations in the direction of streaming are ignored $(k_x = 0)$ and for the potentially unstable case, the system is unstable whether the kinematic viscoelasticity is greater than or less than medium permeability divided by medium porosity. However, for the special case when perturbations in the direction of streaming are ignored $(k_x = 0)$ and for the potentially stable case, the system is stable or unstable according as kinematic viscoelasticity is less than or greater than medium permeability divided by medium porosity. This is in contrast to the special case when perturbations in the directions of streaming are ignored $(k_x = 0)$ and for the Rivlin-Ericksen viscoelastic fluid (as well as Newtonian fluid), the system is stable for potentially stable configuration and is unstable for the potentially unstable configuration.

It is also clear from equation (19), that for the special case when perturbations in the directions of streaming are ignored $(k_x = 0)$, the perturbations transverse to the direction of streaming $(k_x \neq 0)$ are unaffected by the presence of streaming.

(b) In every other direction, instability occurs when

$$\frac{\alpha_1\alpha_2 k_x^2}{\varepsilon^2}(U_1 - U_2)^2\left(1 - \frac{\varepsilon\nu'}{k_1}\right) > gk(\alpha_1 - \alpha_2)\left(1 - \frac{\varepsilon\nu'}{k_1}\right) \quad . \tag{24}$$

Again the kinematic viscosities ν_1 and ν_2 and the kinematic viscoelasticities ν_1' and ν_2' , of two fluids here are assumed to be equal (let $\nu_1 = \nu_2 = \nu$, $\nu_1' = \nu_2' = \nu'$, say) but this simplifying assumption does not obscure any of the essential features of the problem.

Thus for a given difference in velocity $(U_1 - U_2)$ and for a given direction of the wave vector $\vec{k}$, instability occurs for all wave numbers

$$k > \left[\frac{g\varepsilon^2(\alpha_1 - \alpha_2)}{\alpha_1\alpha_2(U_1 - U_2)^2 cos^2\theta}\right], \tag{25}$$

where θ is the angle between the direction $\vec{k}(k_x, k_y, 0)$ and $\vec{U}(U, 0, 0)$ $i.e\ k_x = k\cos\theta$. Hence, for a given velocity difference $(U_1 - U_2)$, instability occurs for the least wave number when $\vec{k}$ is in the direction of $\vec{U}$ and this minimum wave number, k_{min} is given by

$$k_{min} = \frac{g\varepsilon^2(\alpha_1 - \alpha_2)}{\alpha_1\alpha_2(U_1 - U_2)^2} \quad . \tag{26}$$

For $> k_{min}$, the system is unstable. For non-porous medium $(\varepsilon \to 1)$, the result reduces to the one obtained by Kelvin for Newtonian fluids and reported by Chandrasekhar [3].

4.2. Case II: In the Presence of Effective Interfacial Tension

In the presence of 'effective interfacial tension' in accordance with (18), there will be stability if

$$\left(1 - \frac{\varepsilon\nu'}{k_1}\right)\frac{k^2\alpha_1\alpha_2(U_1 - U_2)^2}{\varepsilon^2} < gk\left(1 - \frac{\varepsilon\nu'}{k_1}\right)\left[(\alpha_1 - \alpha_2) + \frac{k^2 T_s}{g(\rho_1 + \rho_2)}\right]. \tag{27}$$

Here we have set $k_x = k$ as these are the disturbances which are most sensitive to Kelvin-Helmholtz instability. Rewriting the preceding inequality (27) as

$$\frac{\alpha_1\alpha_2(U_1 - U_2)^2}{\varepsilon^2} < g\left[\frac{(\alpha_1 - \alpha_2)}{k} + \frac{kT_s}{g(\rho_1 + \rho_2)}\right] \quad . \tag{28}$$

The right-hand side (RHS) of this inequality (28) has a minimum when

$$\frac{d}{dk}\left[g\left\{\frac{(\alpha_1 - \alpha_2)}{k} + \frac{kT_s}{g(\rho_1 + \rho_2)}\right\}\right] = 0$$

i.e. when

$$-\frac{(\alpha_1 - \alpha_2)}{k^2} + \frac{T_s}{g(\rho_1 + \rho_2)} = 0$$

or when

$$\frac{(\alpha_1 - \alpha_2)}{k^2} = \frac{T_s}{g(\rho_1 + \rho_2)} \quad . \tag{29}$$

If we denote the value of k, given by equation (29) by k^* there will be stability if

$$(U_1 - U_2)^2 < \frac{g\varepsilon^2}{\alpha_1\alpha_2}\left[\frac{(\alpha_1 - \alpha_2)}{k^*} + \frac{(\alpha_1 - \alpha_2)}{k^*}\right],$$

i.e. if

$$(U_1 - U_2)^2 < \frac{2g\varepsilon^2}{k^*}\frac{(\alpha_1 - \alpha_2)}{\alpha_1\alpha_2} . \tag{30}$$

Inserting the value of k^* in accordance with equation (29) i.e.

$$k^* = \sqrt{\frac{g(\rho_1 - \rho_2)}{T_s}} \ ,$$

we conclude that the 'effective interfacial tension' will suppress the Kelvin-Helmholtz instability if

$$(U_1 - U_2)^2 < 2g\varepsilon^2 \frac{(\alpha_1 - \alpha_2)}{\alpha_1\alpha_2} \sqrt{\frac{T_s}{g(\rho_1 - \rho_2)}} \ ,$$

i.e. if

$$(U_1 - U_2)^2 < \frac{2\varepsilon^2}{\alpha_1\alpha_2} \sqrt{\frac{T_s g(\alpha_1 - \alpha_2)}{(\rho_1 + \rho_2)}} \ . \qquad (31)$$

The streaming of Walters B′ viscoelastic fluids in porous medium has the same result (31) as that for the streaming of Newtonian fluids in porous medium [24], inequality (34), p. 1394). To illustrate the importance of the above result in petroleum engineering, we consider the streaming of oil over a glycerine-water mixture through sandstone. Here $\rho_2 = 0.875\, g/cm^3$ is taken as the density of oil, $\rho_1 = 1.21\, g/cm^3$ as the density of the glycerine-water mixture, $g = 981\, cm/s^2$, $T_s = 33\, dyne/cm$ and porosity $\varepsilon = 0.10$ (say), whence $\alpha_1 = 0.58$ and $\alpha_2 = 0.42$. Inequality (31) predicts that there will be stability for $|U_1 - U_2| < 2.02\ cm/s$

When $|U_1 - U_2|$ has this maximum velocity compatible with stability, we find

$$k^* = 3.15\, cm^{-1}, \lambda^* = 2\pi/k^* = 1.99\, cm, \quad n^* = \alpha_2 k^* |U_1 - U_2| = 2.67\, s^{-1},$$
$$n^*/k^* = 0.85\ cm/s \ .$$

Therefore, when $|U_1 - U_2|$ exceeds $2.02\ cm/s$, instability will manifest itself as surface waves with wavelength $1.99\ cm$ and wave velocity $0.85\ cm/s$.

The demonstration and verification of the illustration (instability of streaming of oil over a glycerine-water mixture through sandstone, with values of various quantities considered here) forms a strong basis for experimental verification/numerical simulation. This has great relevance and importance in immiscible and miscible displacements of oil by water/air/gas-, in the petroleum industry.

The 'Effective interfacial tension' is able to suppress this Kelvin-Helmholtz instability for small wavelength perturbations. The medium porosity reduces the stability range given in terms of a difference in streaming velocities. For non-porous medium $(\varepsilon \to 1, k_1 \to \infty)$, equation (31) yields the result due to Kelvin (cf. Chandrasekhar [3], inequality (40), p. 486), implying that the surface tension (or 'interfacial tension') completely suppresses the Kelvin-Helmholtz instability for small wavelengths. To illustrate the importance of this result in oceanography, we consider the streaming of air over sea-water $(\rho_1 = 1.02\ g/cm^3, T = 74\ dynes/cm,\ g = 981\, cm/s^2, \alpha_2 = 0.00126)$, Kelvin's result predicts that there will be stability for

$$|U_1 - U_2| < 650\, cm/s \ .$$

When $|U_1 - U_2|$ has this maximum stable value compatible with stability, one finds

$$k^* = 3.68\ \ cm^{-1}$$
$$\lambda^* = 2\pi/k^* = 1.71\ \ cm$$
$$n^* = \alpha_2 k^* |U_1 - U_2| = 3.02\, s^{-1}$$
$$and\ \ n^*/k^* = 0.82\ \ cm/s \ .$$

Hence, when $|U_1 - U_2|$ exceeds 650 cm/s (12.5 nautical miles per hour) instability will manifest itself as surface waves with wavelength 1.71 cm and wave velocity 0.82 cm/s. Lord Kelvin, thus, predicted a wind speed of 650 cm/s for the onset of Kelvin-Helmholtz instability. Munk [27], has pointed out that a wind speed of 650 cm/s is associated with several phenomena observed on the surface of the seas. The experimental demonstration by Francis [4] of an unstable wave produced by air blown over viscous oil in a wind tunnel, is in close conformity with the theoretically predicted wind speed of 650 cm/s by Kelvin [2].

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