



Prospective Simulations of Brazilian Household Air Conditioning Energy Consumption and Carbon Emission

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Abstract

Brazil presents a flourishing market for environmental cooling due to its geographic position, increasing global warming and the growing income of its population. In this work three different types of household appliance are considered to meet the cooling environmental needs in the 2014-2026 time horizons. The main concern of the work was to show the different scenarios supplying these needs from a non-linear methodological perspective, which may indicate some possible undesirable paths.

Keywords: Forecasting; Competition; Air conditioners; Residential; Scenarios.

1. Introduction

Brazil is a striking example of *tropicaliness*. With a territorial extension of 8,513,844 square kilometers its latitude ranges from 5° 16' (north latitude) to 33° 45' (south latitude) with average annual temperatures exceeding 18° in 95% of its territory. This thermal behavior apparently has a rising trend. A study carried out in the city of São Paulo, Brazil, showed that in 1920 the average annual temperature was 17.7° C, whereas today this average temperature is around 19° C [1]. Another study performed by Stensjö [2], shows an increase in CDD (Cooling Degree Day) and reduction of HDD (Heating Degree Day) in the period 1961-1990 for Brazil. Such bias was also detected worldwide by the NASA [3] study, which shows a clear trend of growth in average temperatures on earth. The reasons for the global warming are not unanimously accepted by scientists, who are basically divided into two groups: those who believe in anthropogenic reasons (greenhouse effect due to GHG emissions by human being) and those who believe in natural process cycles associated with the earth's own thermal dynamic structure. The fact is that this global warming trend is real and has impacting consequences on the way of life and resources of mankind. An obvious impact is the increase of energy consumption and CO₂ emission [4, 5] in order to maintain the thermal comfort through a sort of air conditioners devices. This behavior can lead to a positive feedback process, that is, the higher the temperature, the higher the energy consumption and, therefore, the higher the GHG emission leading to a further increase in temperature, if the supporters of this explanation are correct. A review article on the household energy consumption and carbon emissions can be found in Oladokun and Odesola [6].

Ownership of the air conditioner in developed countries and mature markets is near saturation, while in developing countries, how is the Brazilian case, there is a strong growth in the last years, due to household income growth and climate change [7, 8]. The income effect on air conditioning sales aside from the climate change influence can be roughly perceived in the last two decades in Brazil. In fact Brazilian per capita GDP in 2009 was 10,420 US\$ (2013) rising to 11,450 US\$ (2013) in 2013, IPEA [9], meanwhile sales of residential air conditioners were R\$ 2.8x10⁹ in 2010 rising to R\$ 4.4x10⁹ in 2014, [10]. Therefore, a large increase in residential air conditioning sales is still expected in the next few years, induced both from climate change and rising population incomes. According to the Ten Year Energy Expansion [11], in 2014 there were 41 units of air conditioners per 100 urban dwellings and the projection for 2026 is 60 units for every 100 urban dwellings. This means, according to IBGE [12] projections, that in 2014 there were about 11.304 x 10⁶ appliances and the projection for 2026 is 20.46 x 10⁶ air conditioners devices (in this work called BRS - Basic Reference Scenario). The various types of equipment on the current market Arora [13], Carson [14], were grouped in the present work, under performance (COP – Coefficient of Performance) point of view, into three categories: a) window and split fixed rotation – Type A (average COP = 2.7 or 665 kWh/appliance/year); b) split Inverter variable rotation – Type B (COP = 3.2 or 561.1 kWh/appliance/year) and c) ultramodern split device – Type C (COP = 4.0 or 448.9 kWh/appliance/year).

The objective of this paper is to evaluate competition scenarios in which these three types of appliances can fill the BRS over the years 2014-2026 and see the related energy consumption and environmental impacts related to them.

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2. Base year (2014) Household Air Conditioner Market Characterization

The 11.304 million household devices in 2014 have consumed about 653 kWh/appliance or a total of 7.4 TWh [11] and were allotted through Brazilian country in the following regions [ABRAVA \[10\]](#): SOUTHEAST (32%), CENTRAL WEST (21%), NORTHEAST (19%), SOUTH (17%) and NORTH (11%). In the 2014 year, the majority of this residential equipment is window type and fixed rotation (type A - 90%) with an average performance coefficient (COP) of 2.7. The remaining 10% are of variable rotation Inverter type B, which COP was assumed equal 3.2 [10]. For the sake of comparison Japan in 2000 had 81.5 million household air conditioning units, with a consumption of 728.8 kWh per appliance, giving a total of 59.4 TWh for cooling and heating [15].

2.1. Service and Products Competition/Substitution in the Market

For Schumpeter, the soul of *capitalism* is *innovation*, which in turn means the offer of a new product/service or will replace the old one in the market. In the Introduction of [Schumpeter \[16\]](#) book, Stiglitz J. says: “*Schumpeterian competition replaced competition in the market with competition for the market. If competition were perfect, innovators would not be able to appropriate any returns to their innovations, and without innovations, economies would stagnate*”. This would be the case for two athletes fighting in a competition and reaching a break even, which would represent an unstable equilibrium point. Fortunately for capitalism, this event ordinarily has a small probability of occurrence.

The current literature on the phenomenon of product substitution or product competition is immense. To verify this, just click on Google Scholar the corresponding keywords. See, for example [Shin \[17\]](#) for a review. The reasons by which the consumer substitutes a given product for another has its origin in a multiple drivers [18]: (1) opportunities (e.g., new products), (2) external restrictions (e.g., stockouts), and (3) internal restrictions (e.g., a budget). For these issues see also [Thomas \[19\]](#), [Porter \[20\]](#), [Lehmann and R. \[21\]](#), [Murray and Häubl \[22\]](#), and. Next to the decision by pricing the environmental aspect has had a great importance in the choice of competing products. For this issue, see [23] With the advent of new sales instruments through Information Technology, there was an extraordinary boost in competitive processes, accelerating the products substitution in the retail sector (see [24-27]).

The combination of all these factors, driven by the phenomenon of *innovation*, determines the replacement/release dynamics of a new product / service in the market.

2.2. Competition/Substitution Dynamical Approach: Non Linear Differential Equations

Mathematical efforts to analyze the competition between two game players have been made since the beginning of the last century, for instance, in the *predator-prey* [Lotka \[28\]](#) and [Volterra \[29\]](#) papers. The basic difficulty to analyze the dynamics of interacting players concerns to the number of competitors: the mathematical problem becomes extraordinary, when this number is larger than *two*. There’s a similar trouble in classical mechanics: the notorious *three body problem*, in which it’s impossible to get an analytical general solutions. The corresponding problems outside physics, in which more than two players are interacting mutually, is out of consideration here.

The employment of non-linear differential equations – NLDE, to explain a wide class of complex dynamical phenomena, has been done not only in the physics and biology fields, but also in economics, medicine and sociology. Approaches for Brazilian economics, utilizing NLDE, were done by these authors in [Kamimura \[30-32\]](#). In the particular case of the biologic competition between two species, several approaches are known. See, for example, [Clark \[33\]](#) and [Boyce and DiPrima \[34\]](#). Some typical examples:

- The Lotka-Volterra *predator-prey* system, above mentioned, in which one of the species (predator) devours the prey. The prey maintenance comes from the environment, which is considered infinite. It is obvious that this NLDE set isn’t suitable to describe the two species competing by the same finite “market”.
- A substitution model developed by [Fisher and Pry \[35\]](#); [Christian and Groscurth \[36\]](#). It assumes that the market shares of the “new” product follows a “logistic” solution and is a simplification of the next case.
- Two “logistic” coupled NLDE, in which two species compete by the same limited resource or market. The corresponding equations are:

$$dN_1/dt = \epsilon_1.N_1 - \sigma_1.N_1^2 - \alpha_{12}.N_1.N_2 \quad \text{Equation 1}$$

$$dN_2/dt = \epsilon_2.N_2 - \sigma_2.N_2^2 - \alpha_{21}.N_1.N_2 \quad \text{Equation 2}$$

The solutions for each of the above equations, if the interaction coefficient $\alpha_{ij} = 0$ (Verhulst equation), are also known by “logistic” or “sigmoid” curve, which behaves exponentially in the beginning due to the $\epsilon_i.N_i$ term; after an enough time interval, both populations N_1 and N_2 reach “saturation” in the $N_{1,2} = (\epsilon_1/\sigma_1$ and $\epsilon_2/\sigma_2)$ values due to the “self-regulation” terms $-\sigma_i.N_i^2$. However, when α_{ij} differs from zero, basically two solutions are observed: the two species living in equilibrium together, when $\alpha_{12} \cdot \alpha_{21} < \sigma_1 \cdot \sigma_2$, or exponential growth of one of the species with the disappearance of the other – the most frequent event. Depending on ϵ_i , σ_i and α_{ij} values, all positives coefficients, a myriad of distinct solutions are possible, but the majority of them ending in the survival of one species and the concomitant vanishing of the competitor. This is a characteristic of nonlinear system: strong dependency in the initial conditions, in which a little advantage of one of the opponents will carry off the other. It is easy to understand why this case is, in general, an expected and frequent solution: the tie condition between two “fighters” isn’t a stable state, even when the “strengthness” of the opponents are almost equivalent. This is what happened, for example, in the famous case of the winner VHS system against the technologic better Betamax system and recently, the Windows versus IBM operational system for PC. These are what [37] called “positive feedbacks” phenomena in the economy. It explains also why “big” companies prefer to make “agreements” concerning its markets than the “open fight” alternative, since it, probably, would carry to the disappearance of one of them. So, the

process of substitution of a given product by a “new” one represents this particular and frequent type of equations 1 and 2 solution and is the focus of this actual paper.

3. Design and Calculation Section

3.1. The Substitution Model Proposed

Although equations 1 and 2 are suitable to represent competition between two opponents, a particular case of substitution “product” process can be written in a simpler mathematical way. In special scenarios studies, when the “product” demand is associated with continuous population or economic growth, the “self-regulation” term $-\sigma_i N_i^2$ is negligible or doesn't have meaning. In addition, if the probability of substitution is high, then the negative sign of equation 2 can be replaced by plus sign. Then, the following equation set can be settled:

$$dN_1/dt = \varepsilon_1 N_1 - \alpha_{12} N_1 N_2 \quad \text{Equation 1.1}$$

$$dN_2/dt = \varepsilon_2 N_2 + \alpha_{21} N_1 N_2 \quad \text{Equation 2.2}$$

The time variation growth of N_1 (displaced product) is proportional to itself and one subtractive term proportional to $N_1 N_2$. Similarly, the time variation of N_2 (substitute product) is proportional to itself *plus* one term proportional to $N_1 N_2$. In this sense ε_i represent the N_1 and N_2 *intrinsic or independent exponential* growth rate, while the α_{ij} represent the substitution coefficients of N_1 by N_2 product. The symmetry (assymetry) of α_{ij} means that all (partial) decaying fraction of N_1 is added to N_2 variation, or in the other words, the service performed by N_1 is substituted by N_2 product, which amount depends on its substitution efficiency. This coefficient represents mathematically, in this reduced model, the sum of all techno-economic and social efforts – trustfulness, competitive prices, technology; use easiness, marketing and fiscal incentives to replace N_1 by N_2 product. In this sense, the knowledge or the estimation of the α_{ij} value is the most important and sensitive point of the numerical analysis.

The equation set 1.1 and 2.2 has only one positive critical point ($N_2 = \varepsilon_1/\alpha_{12}$). For N_2 initial values lesser than this critical point, the further N_2 solutions will be driven to it and subsequently to unlimited growth with the concomitant disappearance of N_1 .

As told before, the set of equations 1.1 and 2.2 is particularly suitable to represent phenomena in which there are no doubt that *a product N_1 will be substituted by a product N_2* . The question, reduced to know “how long” it takes, depends on which differentials the substitute will represent to consumers: lower prices, technological and tax advantages, lower environmental impacts, new marketing internet boom or natural monopole advantage.

The time variation growth of N_1 (displaced appliance) is proportional to itself and one subtractive term proportional to $N_1 \times N_2$. Similarly, the time variation of N_2 (substitute appliance) is proportional to itself *plus* one term proportional to $N_1 \times N_2$. In this sense ε_i ($i=1, 2$) represents the N_1 and N_2 *intrinsic and independent effective* growth rate (independent appliance i growth rate minus i scrap rate), while the α_{ij} represent the substitution coefficients of (i) by (j) appliance. The symmetry of the coefficient α_{ij} means that the decreased fraction of N_1 variation is added to N_2 variation, which was adopted in this present work. Therefore $\alpha_{12} = \alpha_{21} = \alpha$.

The coefficient α_{ij} represents mathematically, in this reduced model, the sum of all techno-economic and social efforts – competitive prices, energy save, technology, durability, environmental appeal, trustfulness, marketing and fiscal incentives to replace N_1 by N_2 appliance.

3.2. 2026 Scenarios Description

Starting 2014 year with 90% type A and 10% with type B appliances (total of 11.304 million) the time evolution to 2026 (total of 20.46 million appliances) market (BRS) will be filled, on a present work, in ten scenarios, as follows:

Scenario 1: $\varepsilon_1=0.05498$; $\varepsilon_2=0$; and $\alpha=0$. Evolution of type A appliance without substitution by B or C type appliance ($\alpha=0$). This scenario means that older devices (type A) will grow at an effective rate of 5.5% per year, and the effective growth rate of new devices (type B or type C) will be zero ($N_2 = \text{constant}$). This scenario unfolds in two: Sc1b and Sc1c.

Scenario 2: $\varepsilon_1=0$; $\varepsilon_2=0.20212$; and $\alpha=0$. Evolution of type A appliance on a constant basis ($N_1 = \text{constant}$), without effective growth rate throughout the period. The effective growth rate of new devices (type B or type C) will be 20.2% by year. There will be no substitution of Type A devices with Type B or Type C devices ($\alpha=0$). This scenario unfolds in two: Sc2b and Sc2c.

Scenario 3: $\varepsilon_1=0.05072$; $\varepsilon_2=0.05072$; and $\alpha=0.018$. Intrinsic and independent effective evolution of type A appliance occurs in the same way with B or C type appliance. A moderate substitution process of A type appliance by B or C type occurs. This scenario unfolds in two: Sc3b and Sc3c.

Scenario 4: $\varepsilon_1=0.05072$; $\varepsilon_2=0.05072$; and $\alpha=0.01$. This scenario is analogous to the previous case, but with a weak substitution process. This scenario unfolds in two: Sc4b and Sc4c.

Scenario 5: $\varepsilon_1=0.03568$; $\varepsilon_2=0.07136$; and $\alpha=0.025$. Strong growth of B or C type appliance – almost double intrinsic and independent effective growth rate than A type. A strong substitution process of A type by B or C type occurs. This scenario unfolds in two: Sc5b and Sc5c.

4. Results

4.1. Final N_1 (type A) and N_2 (type B or C) Results of Scenarios

In the following pictures $N_1(t)$ will be represented by EqA(t) (type A appliance), whereas $N_2(t)$ will be represented by EqBC(t) (B or C type appliance). EqTot(t) is the sum of $N_1(t)$ and $N_2(t)$ for each year (t).

Fig-1. Scenario 1: $\epsilon_1=0.05498$; $\epsilon_2=0$; and $\alpha=0$. The type A appliances grows exponentially at 5.5% by year, whereas the type B or C appliances are kept constant in the period. No substitution occurs between A by B or C type appliances

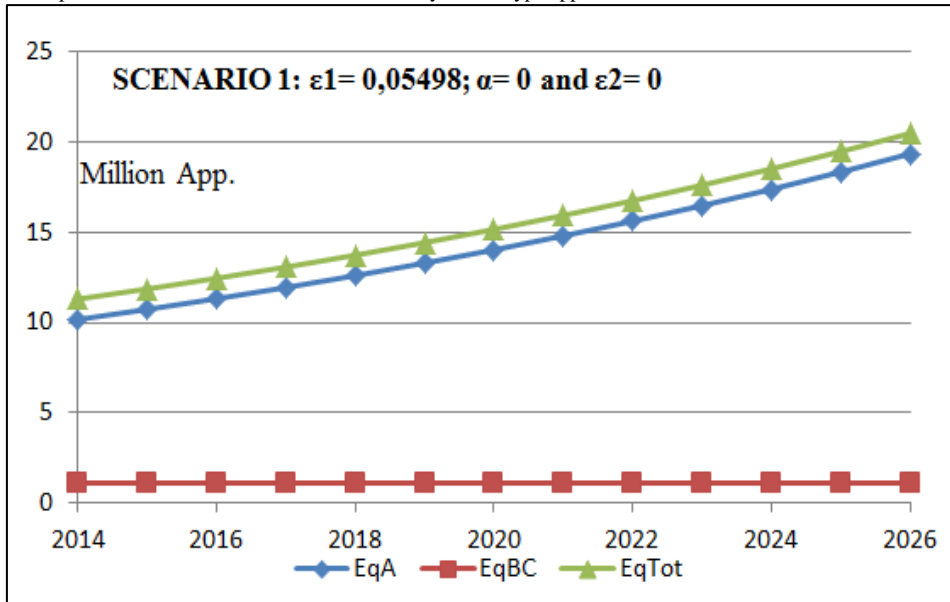


Fig-2. Scenario 2: $\epsilon_1=0$; $\epsilon_2=0.20212$; and $\alpha=0$. Type A appliance is kept constant, meanwhile B or C type grows strongly at 20.2% by year. No substitution occurs between A by B or C type appliances

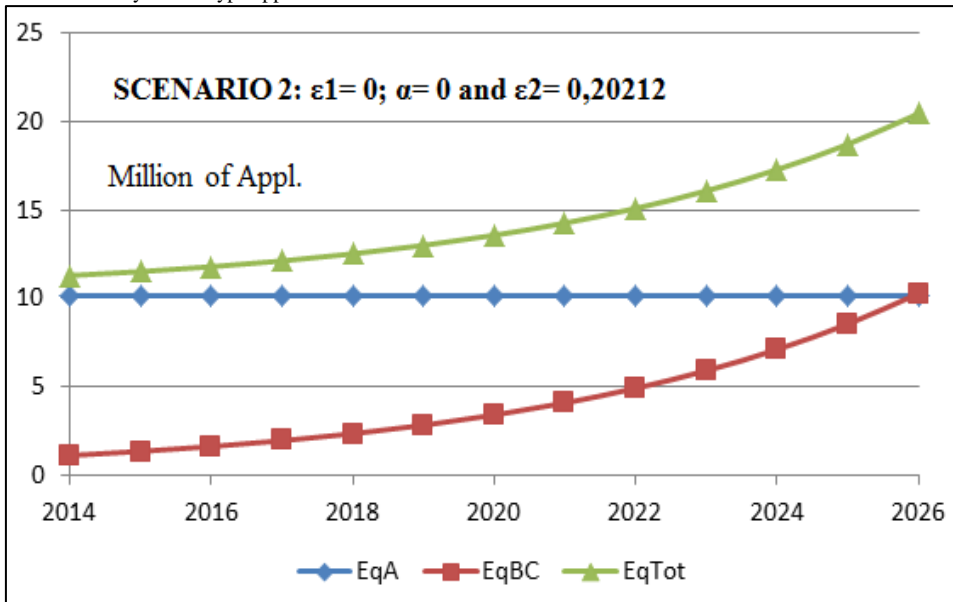


Fig-3. Scenario 3: $\epsilon_1=0.05072$; $\epsilon_2=0.05072$; and $\alpha=0.018$. Intrinsic independent growth rate of A type equal to B or C type. A moderate substitution of A by B or C occurs

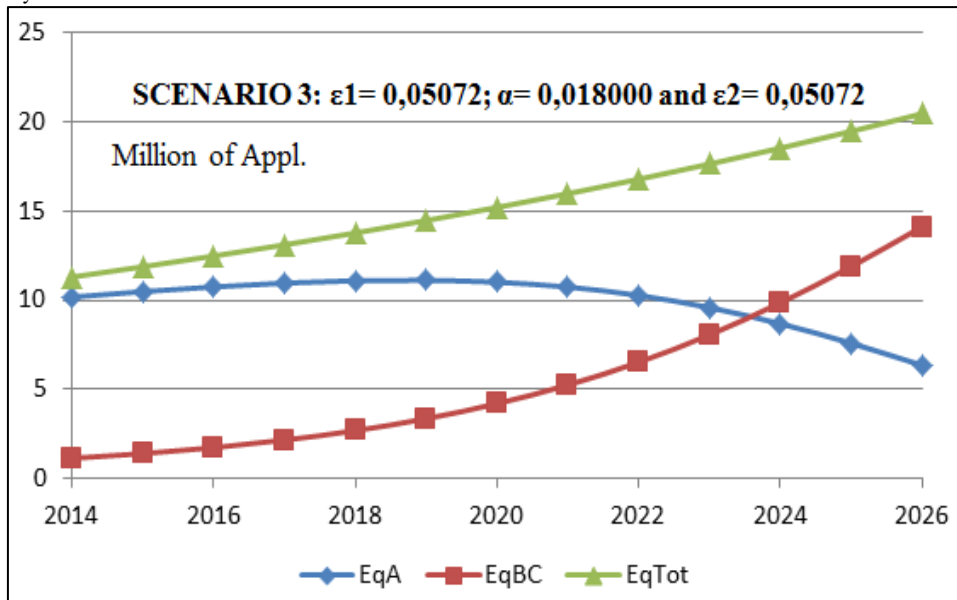


Fig-4. Scenario 4: $\epsilon_1=0.05072$; $\epsilon_2=0.05072$; and $\alpha=0.01$. Analogous to the previous scenario, A weak substitution of A by B or C occurs

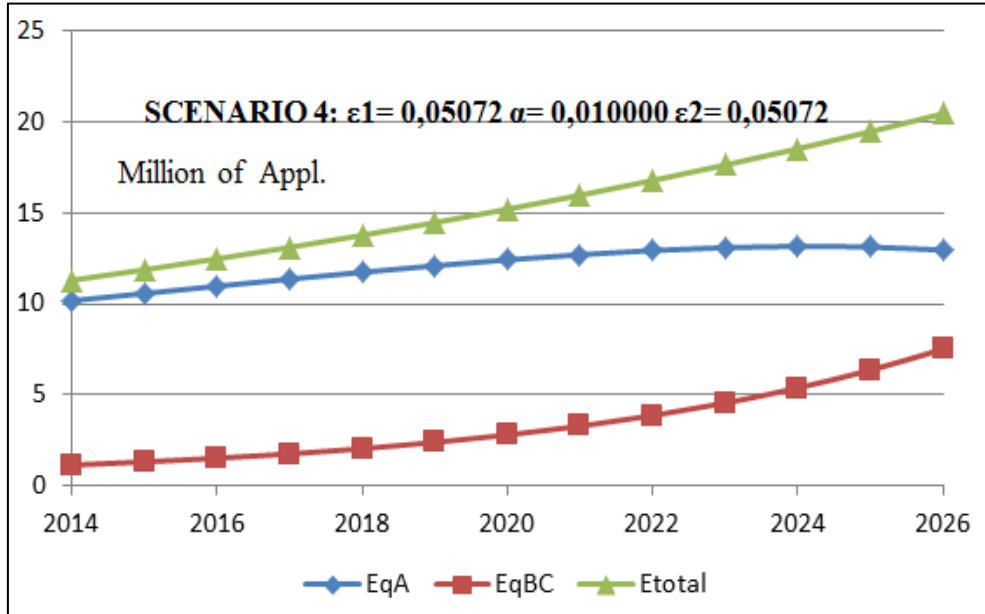


Fig-5. Scenario 5: $\epsilon_1=0.03568$; $\epsilon_2=0.07136$; and $\alpha=0.025$. Strong intrinsic independent growth rate of B or C type (almost double rate of A type intrinsic independent growth rate). Strong substitution of A by B or C occurs

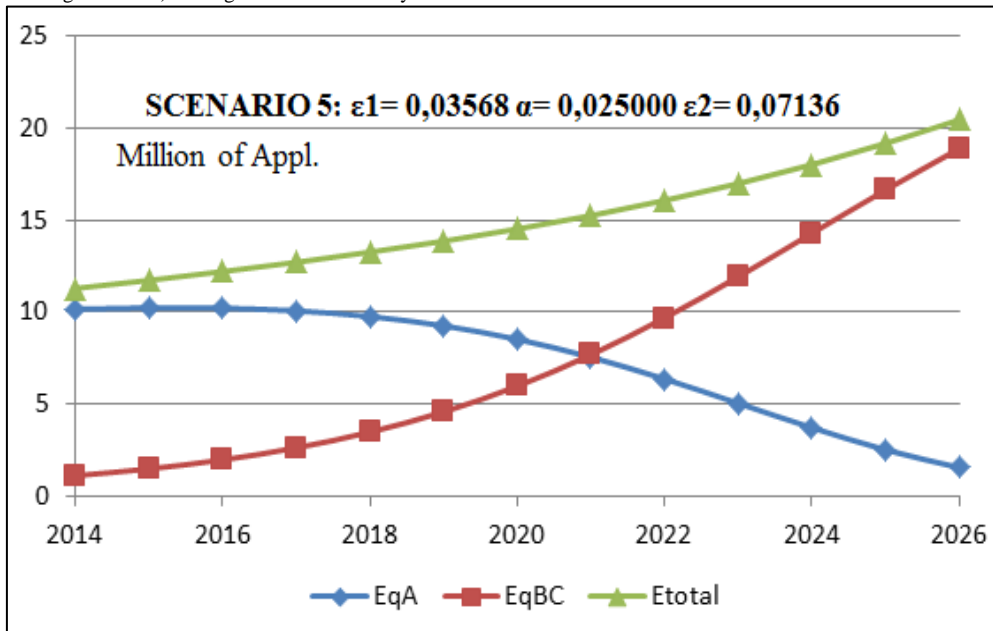


Fig-6. Scenario 3 versus Scenario 4. Total A type substituted $Es_1 = 10.04$ million and $Es_2 = 4.54$ million

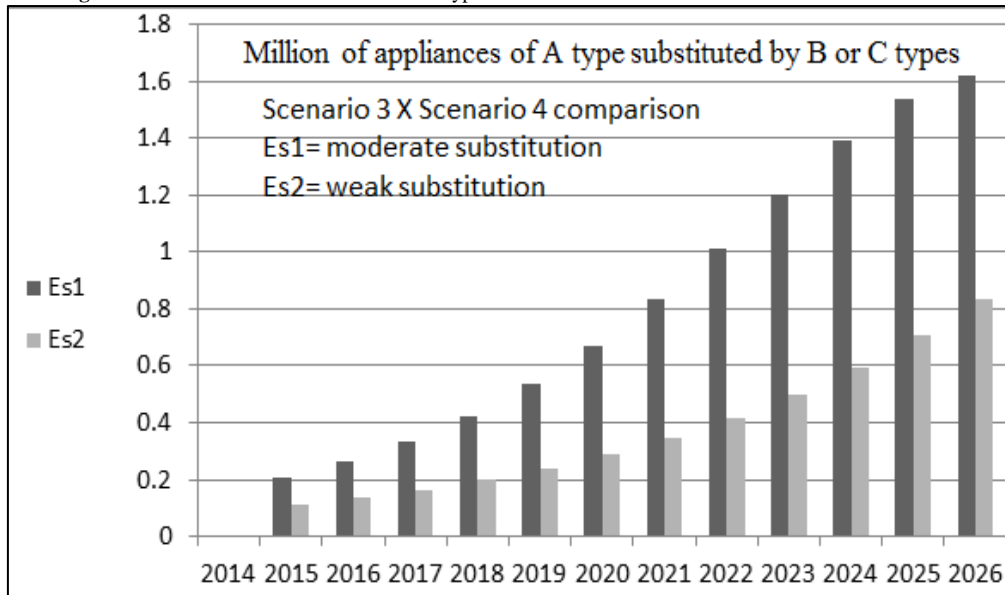


Fig-7. Scenario 3 versus Scenario 5. Total A type substituted Es1 = 10.04 million and Es = 11.94 million

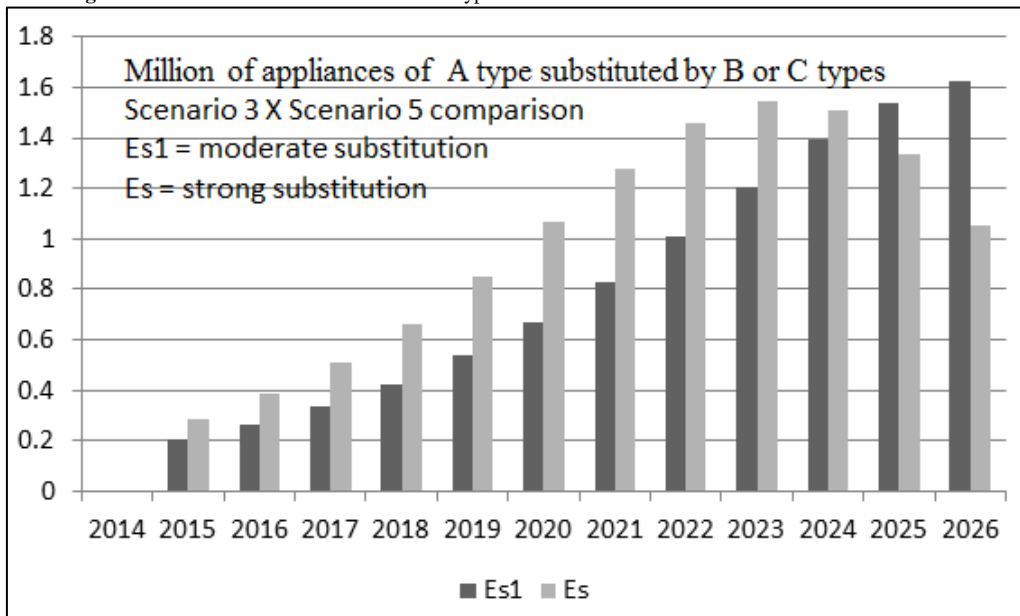


Fig-8. Scenario 3 versus Scenario 5. Total A type substituted Es = 11.94 million and Es2 = 4.54 million

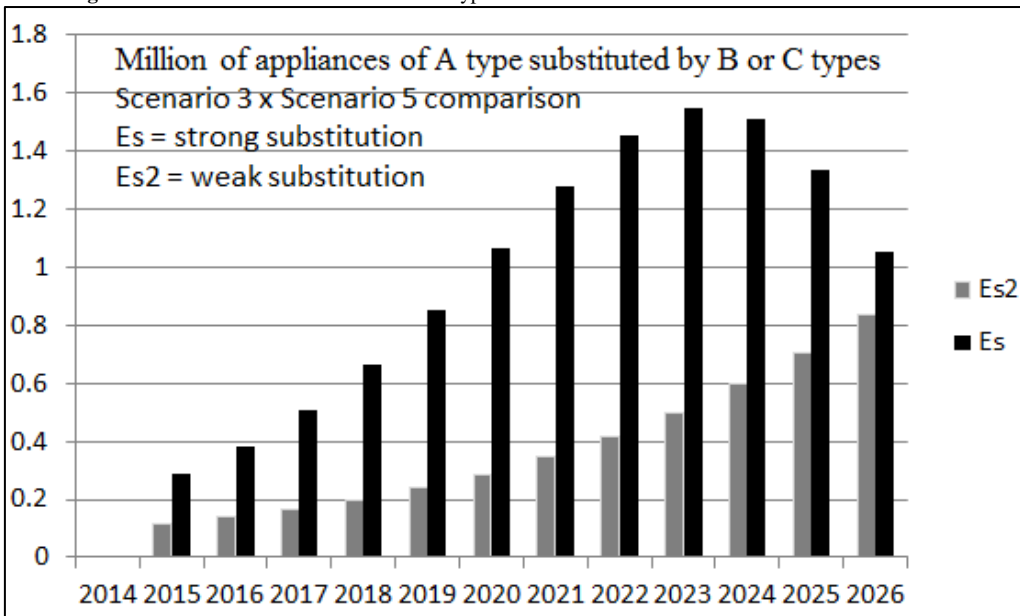


Table 1 shows the numerical results of Equation 1.1 and Equation 2.2, or the time evolution of the number of household air conditioner type A or Type B or C appliances, with previously cited boundary conditions, for Scenarios(i), i = 1 to 5.

It is worth to note in the Fig.7, the resemblance between the total A type appliance substituted in the Scenario 3 (moderate penetration, $\alpha=0.018$) with the Scenario 5 (strong penetration, $\alpha=0.025$), due to non-linear nature of the dynamic process.

Table-1. Appliances number evolution on Sc(i), associated to figures Fig.(i), i=1 to 5

year	Scenario 1			Scenario 2			Scenario 3			Scenario 4			Scenario 5		
	EqA	EqBC	EqTot	EqA	EqBC	EqTot	EqA	EqBC	EqTot	EqA	EqBC	Ettotal	EqA	EqBC	Ettotal
2014	10,2	1,13	11,3	10,17	1,13	11,3	10,17	1,13	11,3	10,17	1,13	11,3	10,17	1,13	11,3
2015	10,7	1,13	11,86	10,17	1,36	11,53	10,48	1,39	11,87	10,57	1,30	11,87	10,25	1,50	11,74
2016	11,3	1,13	12,45	10,17	1,63	11,80	10,75	1,73	12,48	10,97	1,51	12,48	10,23	1,99	12,22
2017	11,9	1,13	13,07	10,17	1,96	12,13	10,96	2,15	13,11	11,36	1,75	13,11	10,08	2,64	12,72
2018	12,6	1,13	13,73	10,17	2,36	12,53	11,09	2,68	13,77	11,74	2,03	13,77	9,78	3,49	13,27
2019	13,3	1,13	14,42	10,17	2,84	13,01	11,12	3,35	14,47	12,09	2,38	14,47	9,27	4,60	13,87
2020	14,0	1,13	15,15	10,17	3,41	13,58	11,01	4,20	15,21	12,42	2,78	15,21	8,54	5,99	14,53
2021	14,8	1,13	15,92	10,17	4,10	14,27	10,74	5,24	15,98	12,70	3,27	15,98	7,57	7,69	15,26
2022	15,6	1,13	16,74	10,17	4,93	15,10	10,27	6,52	16,79	12,93	3,85	16,79	6,38	9,70	16,08
2023	16,5	1,13	17,59	10,17	5,92	16,09	9,58	8,05	17,64	13,09	4,55	17,64	5,06	11,94	17,00
2024	17,4	1,13	18,50	10,17	7,12	17,29	8,68	9,85	18,53	13,16	5,37	18,53	3,73	14,30	18,03
2025	18,3	1,13	19,45	10,17	8,56	18,73	7,58	11,89	19,47	13,12	6,35	19,47	2,53	16,65	19,18
2026	19,3	1,13	20,46	10,17	10,29	20,46	6,34	14,12	20,46	12,95	7,51	20,46	1,57	18,90	20,46

5. Discussions

5.1. Energy and Environmental Results Associated to Scenarios

Table 2 shows the total electricity consumption during the period 2014 - 2026, for household air conditioning appliance in each Sc(i) scenario defined at the beginning. As expected, the largest difference in energy consumption is between scenarios Sc1b and Sc5c, that was 23.38 TWh. corresponding to the maintenance of the effective *status quo* situation in 2014 versus making a big effort to insert more efficient equipment during the period considered. To get an idea of the order of magnitude of this value, this is equivalent to 6.78 million tons of oil consumed in a fuel oil thermoelectric plant with 27.5% efficiency. It is also equivalent to 19% of total Brazilian thermo-electricity produced by non-renewable sources in 2017 [38]. If Sc5c occurs instead of Sc1b will mean that 20.2 million tons of CO₂ will no longer be emitted during the period 2014-2026. This represents 5.3% of all CO₂ Brazilian emissions from non- renewable fuels in 2017 [38].

It is interesting to note that, from the energy consumption point of view, some scenarios are almost equivalent, such as (Sc1b, Sc1c, Sc4b); (Sc5b, Sc2b, Sc3c); and (Sc3b, Sc4c). These energy-related scenarios certainly involve distinct technical, economic, and social efforts in their implementations, so through this kind of calculation we can avoid worthless endeavor.

Table-2. Appliances number evolution on Sc(i), associated to figures Fig.(i), i=1 to 5

Energy Consumption - TWh (2014-2016)				Sc(i)b	Sc(i)c
	(A type)	(B type)	(C type)	Total A+B	Total A+C
Sc1	123,66	8,24	6,72	131,9	130,38
Sc2	87,92	31,21	25,09	119,13	113,01
Sc3	85,63	40,57	32,59	126,2	118,22
Sc4	104,6	24,57	19,78	129,17	124,38
Sc5	63,28	56,4	45,25	119,67	108,52

Table 2. Total energy consumption associated to scenarios Sci, i= 1 to 5. The last two columns refer to the unfolding of each scenario, corresponding to the use of equipment B or C (Scib or Scic).

6. Conclusion

This paper examines the possible scenarios for the evolution of the Brazilian household air conditioners market in 2014 – 2026 time intervals. The main concern of the work was to show the different substitution scenarios market supply from a non-linear methodological point of view. This method allows the planner to simulate and choose the various possible alternatives to achieve a given objective concerned to energy consumption or environmental impacts. In this way, the planner can manage waste of resources avoiding undesirable dynamical paths.

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