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Convection in a Compressible Rotating Viscoelastic Fluid

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Abstract

The aim of the present work was to study the thermosolutal convection in a compressible rotatory Rivlin-Ericksen viscoelastic fluid in permeable media. Following linear stability theory and normal mode analysis method, the dispersion relation is obtained. For the case of stationary convection, the Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid. The compressibility, stable solute gradient and rotation are found to postpone the onset of convection, whereas medium permeability hastens or postpones the onset of convection for the case of stationary convection. Also it is found that the system is stable for $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \le \frac{4\pi^2}{P_1}$ and under the condition $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}$, the system becomes unstable. The case of overstability has also been considered wherein sufficient conditions for the non-existence of overstability are obtained. The stable solute gradient and rotation are found to introduce oscillatory modes in the system.
Keywords: Thermosolutal convection; Compressible viscoelastic fluid; Porous medium; Uniform rotation.

1. Introduction

Thermosolutal convection or more generally double-diffusive convection, like its classical counterpart, namely, single-diffusive convection, has carved a niche for itself in the domain of hydrodynamic stability on account of its interesting complexities as a double-diffusive phenomenon as well as its direct relevance in the fields of Oceanography, Astrophysics, Geophysics, Limnology and Chemical engineering etc. can be seen from the review articles by Turner [1] and Brandt and Fernando [2]. An interesting early experimental study is that of Caldwell [3]. The problem is more complex than that of a single-diffusive fluid because the gradient in the relative concentration of two components can contribute to a density gradient just as effectively as can a temperature gradient. Further, the presence of two diffusive modes allows either stationary or overstable flow states at the onset of convection depending on the magnitude of the fluid parameters, the boundary conditions and the competition between thermal expansion and the thermal diffusion. More complicated double-diffusive phenomenon appears if the destabilizing thermal/concentration gradient is opposed by the effect of a magnetic field or rotation.

Chandrasekhar [4], has studied the theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation. Thermosolutal convection concerns flow that can arise when a layer of fluid with a dissolved solute (such as salt) is heated from below. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. Examples of particular interest are provided by ponds built to trap solar heat [5] and some Antarctic lakes [6]. Veronis [7], has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. Brakke [8], explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason, *et al.* [9], found that this instability, which is deleterious to certain biochemical separation, can be suppressed by rotation in the ultracentrifuge.

The problem of double-diffusive convection in fluids in porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust [10]. Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice- versa. According to McDonnel [11] the physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Stommel and Fedorov [12] and Linden [10] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions.

When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis [13] have simplified the set of equations governing the flow of compressible fluids under the assumptions that (a) the depth of the fluid layer is much less than the scale height, as defined by them, and (b) the fluctuations in temperature, density, and pressure, introduced due to motion; do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and C_v is replaced by C_p. Using these approximations, Sharma [14] has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic field.

Many common materials such as paints, polymers, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. Due to the growing use of these viscoelastic materials in various manufacturing and processing industries, in geophysical fluid dynamics, in chemical technology and in petroleum industry, considerable effort has been directed towards understanding their flow. Oldroyd [15], proposed a theoretical model for a class of viscoelastic fluids. Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies of waves and stability in different viscoelastic fluid dynamical configuration has been carried out by several researchers in the past. The nature of instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. An experimental demonstration by Toms and Strawbridge [16] reveals that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. Sharma and Sharma [17], studied the stability of the plane interface separating two Oldroydian viscoelastic superposed fluids of uniform densities. There are many elastico-viscous fluids that cannot be characterized by Oldroyd`s constitutive relations. Rivlin and Ericksen [18], fluid is one such class of elastico-viscous fluids. It is well known that the Rivlin and Ericksen [18] is characterized by the constitutive equations

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 + \mu^{iii} A_2^2 + \mu^{iv}(A_1 A_2 + A_2 A_1) + \mu^{v}(A_1^2 A_2 + A_2 A_1^2) + \mu^{vi}(A_1 A_2^2 + A_2^2 + A_1) + \mu^{vii}(A_1^2 A_2^2 + A_2^2 A_1^2) \quad (1)$$

where S is the Cauchy stress tensor, 'p' is an arbitrary hydrostatic pressure, I is the unit tensor and μ′s are polynomial functions of the traces of the various tensors occurring in the representation, matrices 'A_1' and 'A_2' are defined by

$$[A_1]_{ij} = (q_{i,j} + q_{j,i}) \quad (2)$$

and

$$[A_2]_{ij} = \frac{\partial [A_1]_{ij}}{\partial t} + q_p [A_1]_{ij,p} + [A_1]_{ip} q_{p,j} + [A_1]_{pj} q_{p,i} \quad (3)$$

'q_p' being velocity vector.

On neglecting the squares and products of 'A_2', we have

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 \quad , \quad (4)$$

where μ, μ^{i} *and* μ^{ii} are three material constants. It is customary to call μ , the coefficient of ordinary viscosity, μ' the coefficient of viscoelasticity *and* μ^{ii}, the coefficient of cross-viscosity. The μ, μ^{i} *and* μ^{ii} are general functions of temperature and material properties. For many fluids such as aqueous solution of polycrylamid and poly-isobutylene, μ, μ^{i} and μ^{ii} may be taken as constants. Such and other polymers are used in the manufacture of parts of spacecrafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, adhesives, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. Sharma and Kumar [19], have studied the hydromagnetic stability of two Rivlin-Ericksen elastico-viscous superposed conducting fluids. Kumar and Singh [20], have studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar, *et al.* [21] have studied the hydrodynamic and hydomagnetic stability of two stratified Rivlin-Ericksen elastic-viscous superposed fluids. The medium has been considered to be non-porous in all the above studies.

The flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in a book by Philips [22]. The gross effect when the fluid slowly percolates through the pores of the rock is given by Darcy`s law. As a result, the usual viscous term in the equations of fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu + \mu' \frac{\partial}{\partial t}\right)\vec{q}\right]$, for Rivlin-Ericksen elastico-viscous fluid, where μ and μ' are the viscosity and viscoelasticity of the fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. The stability of superposed Rivlin-Ericksen elastic-viscous fluids permeated with suspended particles in a porous medium has been considered by Kumar [23]. Kumar, *et al.* [24], have studied the instability of two rotating viscoelastic (Rivlin-Ericksen) superposed fluids with suspended particles in a porous medium. In another study, Kumar, *et al.* [25] have considered the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids through a porous medium. Aggarwal and Dixit [26], have considered the thermosolutal instability of a layer of elastico-viscous fluid permeated with suspended particles in porous medium under the effect of compressibility.

Keeping in mind the importance of flow through porous media in geophysics, soil sciences, ground-water hydrology, atmospheric physics, astrophysics, and various applications mentioned above, our interest, in the present paper, is to bring out rotation effect on thermosolutal convection in a compressible Rivlin-Ericksen elastico-viscous fluid through porous medium. The analysis of the present work begins with Section 2, which formulates the problem for Rivlin-Ericksen viscoelastic compressible fluid by using the Boussinesq approximation, Spiegel and Veronis [13] assumptions, linearized theory, and perturbation theory. In Section 3, a dispersion relation is obtained by using the normal mode technique. The effects of compressibility, rotation, stable solute gradient and medium permeability for the case of stationary convection are discussed analytically in Section 4. In Section 5, some important theorems related to stability of the system are discussed and further sufficient conditions for the non-existence of overstability are also obtained.

2. Formulation of the Problem and Perturbation Equations

Here we consider an infinite, horizontal, compressible Rivlin-Ericksen elastico-viscous fluid layer of thickness d in a porous medium, heated and soluted from below so that the temperatures, densities, and solute concentrations at the bottom surface $z = 0$ are T_0, ρ_0, and C_0, and at the upper surface $z = d$ are T_d , ρ_d, and C_d, respectively, with the z −axis being taken as vertical, and that a uniform adverse temperature gradient ($\beta = |dT/dz|$) and a uniform solute gradient ($\beta' = |dC/dz|$) are maintained. This layer is acted on by a gravity field $\vec{g}(0,\ 0,\ -g)$ and rotation $\vec{\Omega}\ (0, 0, \Omega)$.

Spiegel and Veronis [13], defined f as any one of the state variables (pressure p, density ρ or temperature T) and expressed these in the form

$$f(x,\ y,\ z,\ t) = f_m + f_0(z) + f'(x,\ y,\ z,\ t) \tag{5}$$

where f_m is the constant space average of f, f_0 is the variation in the absence of motion and f' is the fluctuation resulting from motion.

The initial state is, therefore, a state in which the density, pressure, temperature, solute concentration and velocity at any point in the fluid are given by

$\rho = \rho(z),\ p = p(z),\ T = T(z),\ C = C(z),\ \vec{q} = 0$ respectively,

where $T(z) = T_0 - \beta z,\ C = C_0 - \beta' z,\ p(z) = p_m - g\int_0^z(\rho_m + \rho_0)dz,$

$$\rho(z) = \rho_m[1 - \alpha_m(T - T_m) + \alpha'_m(C - C_m) + K_m(p - p_m)],$$
$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_m (= \alpha, say), \alpha'_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial C}\right)_m (= \alpha', say),$$
$$K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m . \tag{6}$$

Let $\delta\rho, \delta p, \theta, \gamma,$ and $\vec{q}(u, v, w)$ denote, respectively, the perturbations in density ρ, pressure p, temperature T, solute concentration C and velocity $\vec{q}(0, 0, 0)$. Then the linearized perturbation equations relevant to the problem are

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p + \vec{g}\frac{\delta\rho}{\rho_m} - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{2}{\varepsilon}(\vec{q} \times \vec{\Omega}), \tag{7}$$

$$\nabla.\vec{q} = 0, \tag{8}$$

$$E\frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)w + \kappa\nabla^2\theta\ , \tag{9}$$

$$E'\frac{\partial\gamma}{\partial t}=\beta' w+\kappa'\nabla^2\gamma\,. \qquad (10)$$

Here $\nu,\nu',c_p,\kappa\ and\ \kappa'$ stand for kinematic viscosity, kinematic viscoelasticity, specific heat at constant pressure, thermal diffusivity and solute diffusivity respectively.

$$E=\varepsilon+(1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c}$$

where ρ_s,c_s and ρ_0,c are the densities and specific heats of the solid (porous matrix) and fluid respectively. E' is a constant analogous to E but corresponding to the solute rather to the heat.

The equation of state is

$$\rho=\rho_m[1-\alpha(T-T_0)+\alpha'(C-C_0)]\,, \qquad (11)$$

where α is the coefficient of thermal expansion and α'analogous the solute coefficient. The suffix zero refers to the values at the reference level $z=0$. The change in density $\delta\rho$, caused by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho=-\rho_m(\alpha\theta-\alpha'\gamma). \qquad (12)$$

Writing the scalar components of equation (7), eliminating $u,v,$ and δp between them by using equations (8) – (10), we obtain

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w=g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-g\alpha'\left(\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}, \qquad (13)$$

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\varsigma=\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}\right], \qquad (14)$$

$$\left(E\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\left(\beta-\frac{g}{c_p}\right)w\,, \qquad (15)$$

$$\left(E'\frac{\partial}{\partial t}-\kappa'\nabla^2\right)\gamma=\beta' w\,, \qquad (16)$$

where $\varsigma=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$, denote the z-component of vorticity.

Consider the case in which both the boundaries are free and temperatures, solute concentrations at the boundaries are kept constant. Then the boundary conditions appropriate to the problem are

$$w=\frac{\partial^2 w}{\partial z^2}=0\,,\theta=0\,,\gamma=0\,,\frac{\partial\tau}{\partial z}=0\ at\ z=0\ and\ z=d. \qquad (17)$$

3. Dispersion Relation

Here we analyze the disturbances into normal modes and assume that the perturbation quantities are of the form

$$[w,\theta,\varsigma,\gamma]=[W(z),\Theta(z),Z(z),\Gamma(z)]e^{ik_x x+ik_y y+nt}\,, \qquad (18)$$

where $k_x\,,k_y$ are wave numbers along $x-$ and $y-$directions respectively, $k\left(=\sqrt{k_x^2+k_y^2}\right)$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (18), equations (13) – (16) in non-dimensional form become

$$\frac{\sigma}{\varepsilon}(D^2-a^2)W+g\frac{a^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma)+\frac{1}{P_1}(1+F\sigma)(D^2-a^2)W+T_A{}^{1/2}dDZ=0, \qquad (19)$$

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}(1+F\sigma)\right)Z=\frac{1}{d}T_A{}^{1/2}DW, \qquad (20)$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{G-1}{G}\right)\frac{\beta d^2}{\kappa}W\,, \qquad (21)$$

$$(D^2-a^2-E'q\sigma)\Gamma=-\frac{\beta' d^2}{\kappa'}W\,, \qquad (22)$$

where

$$a=kd,\sigma=\frac{nd^2}{\nu},x=x^*d,y=y^*d\,,z=z^*d\ and\ D=\frac{d}{dz^*}\,,p_1=\frac{\nu}{\kappa}\ is\ the\ Prandtl\ number,$$

and $q=\frac{\nu}{\kappa'}$ is the Schmidt number, $P_1=\frac{k_1}{d^2}$ is the dimensionless medium permeability, $F=\frac{\nu'}{d^2}$ is the dimensionless kinematic viscoelasticity and $G=\frac{c_p\beta}{g}$ is the dimensionless compressibility parameter. We shall suppress the star $(*)$ for convenience hereafter.

Eliminating $Z,\Theta,and\ \Gamma$ between equations (19) – (22), we obtain

$$\begin{aligned}(D^2-a^2)(D^2-a^2-Ep_1\sigma)(D^2-a^2-E'q\sigma)\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}(1+F\sigma)\right]^2 W\\ =\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}(1+F\sigma)\right]\left[Ra^2\left(\frac{G-1}{G}\right)(D^2-a^2-E'q\sigma)W-Sa^2(D^2-a^2-Ep_1\sigma)W\right]\\ -T_A(D^2-a^2-Ep_1\sigma)(D^2-a^2-E'q\sigma)D^2W\,, \qquad (23)\end{aligned}$$

where $R=\frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S=\frac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the solute Rayleigh number and $T_A=\frac{4\Omega^2d^4}{\varepsilon^2\nu^2}$ is the Taylor number.

Using expression (18), the boundary conditions (17), in non-dimensional form, transform to

$$W = D^2 W = 0\,, \Theta = 0\,, \Gamma = 0\,, DZ = 0 \; at \; z = 0 \; and \; z = 1. \tag{24}$$

Using the boundary conditions (24), it can be shown with the help of equations (19) – (22) that all the even order derivatives of W must vanish at $z = 0$ and $z = 1$. Hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z\,, \tag{25}$$

where W_0 is a constant. Substituting the proper solution (25) in equation (23), we obtain the dispersion relation

$$R_1 = \frac{\frac{G}{G-1}\left[\begin{array}{c}(1+x)(1+x+iEp_1\sigma_1)(1+x+iE'q\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1+i\sigma_1\pi^2F)\right\}^2 \\ +S_1x(1+x+iEp_1\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1+i\sigma_1\pi^2F)\right\}+T_1(1+x+iEp_1\sigma_1)(1+x+iE'q\sigma_1)\end{array}\right]}{x(1+x+iE'q\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1+i\sigma_1\pi^2F)\right\}}, \tag{26}$$

where

$$R_1 = \frac{R}{\pi^4}\,, S_1 = \frac{S}{\pi^4}\,, x = \frac{a^2}{\pi^2}\,, P = \pi^2 P_1\,, i\sigma_1 = \frac{\sigma}{\pi^2} \; and \; T_1 = \frac{T_A}{\pi^4}.$$

4. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (26) reduces to

$$R_1 = \frac{G}{G-1}\left[\frac{(1+x)^2}{xP} + S_1 + \frac{T_1(1+x)P}{x}\right], \tag{27}$$

which expresses modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters G, S_1, T_1 and P. For fixed P, T_1, S_1 and G (accounting for compressibility effect), we find that

$$\bar{R}_c = \left(\frac{G}{G-1}\right) R_c\,, \tag{28}$$

where $\bar{R}_c$ and R_c denote respectively the critical Rayleigh numbers in the presence and absence of compressibility. $G > 1$ is relevant here. The cases $G < 1$ and $G = 1$ correspond to negative and infinite values of the critical Rayleigh numbers in the presence of compressibility, which are not relevant in the present study. The effect of compressibility is thus to postpone the onset of thermosolutal convection. Hence compressibility has a stabilizing effect.

It is evident from equation (27) that for stationary convection, Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. Equation (27) yields

$$\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right)\left[-\frac{(1+x)^2}{xP^2} + \frac{T_1(1+x)}{x}\right], \tag{29}$$

$$\frac{dR_1}{dS_1} = \frac{G}{G-1}\,, \tag{30}$$

and

$$\frac{dR_1}{dT_1} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{x}P\,, \tag{31}$$

which shows that stable solute gradient and rotation postpone the onset of convection.

When

$$\frac{1}{P^2}\frac{(1+x)^2}{x} < or > \frac{T_1(1+x)}{x}$$

the medium permeability hastens or postpones the onset of convection which makes the system stable or unstable.

The critical Rayleigh number is derived from equation (27) by putting $\frac{dR_1}{dx} = 0$, i.e.

$$\left(\frac{G}{G-1}\right)\left[\frac{x^2-1}{x^2P} - \frac{T_1P}{x^2}\right] = 0$$

or

$$x^2 = 1 + T_1P^2\,. \tag{32}$$

with $x = \sqrt{1+T_1P^2}$, as wave number is always positive, equation (27) will yield the required critical Rayleigh number.

5. Some Important Theorems

Theorem 1: The system is stable for $G < 1$.

Proof: Multiplying equation (19) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (20) – (22) together with boundary conditions (24), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1}(1+F\sigma)\right] I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}(I_2 + Ep_1\sigma^* I_3) + \frac{g\alpha'\kappa' a^2}{\nu\beta'}(I_4 + E'q\sigma^* I_5) + d^2\left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_1}(1+F\sigma^*)\right) I_6 = 0\,, \tag{33}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz\,, \quad I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz, \quad I_3 = \int_0^1 (|\Theta|^2)dz,$$

$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)dz\,, \quad I_5 = \int_0^1 (|\Gamma|^2)dz\,, \quad I_6 = \int_0^1 (|Z|^2)dz, \tag{34}$$

where σ^* is the complex conjugate of σ and the integrals $I_1 - I_6$ are all positive definite.

Putting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of equation (33), we get

$$\sigma_r\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5 + d^2\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_6\right]$$
$$= -\left[\frac{1}{P_1}I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}I_2 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4 + \frac{d^2}{P_1}I_6\right], \tag{35}$$

and

$$\sigma_i\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 + \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3 - \frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5 - d^2\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_6\right] = 0. \tag{36}$$

It is evident from equation (35) that if $G < 1$, σ_r is negative meaning thereby the stability of the system.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to the case of non-rotatory field and in the absence of stable solute gradient where modes are non-oscillatory, for $G > 1$.

Proof: Equation (36) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of stable solute gradient and rotation, equation (36) gives

$$\sigma_i\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 + \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3\right] = 0, \tag{37}$$

and the terms in brackets are positive definite when $G > 1$. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied. The presence of each, the stable solute gradient and rotation introduce oscillatory modes in the system, which were non-existent in their absence.

Theorem 3: The system is stable for $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \le \frac{4\pi^2}{P_1}$ and under the condition $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}$, the system becomes unstable.

Proof: From equation (36) it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, equation (35) upon utilizing (36) and the Rayleigh-Ritz inequality gives

$$\left[\frac{4\pi^2}{P_1} - \frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu}\right]\int_0^1 |W|^2dz + \frac{\pi^2+a^2}{a^2}\left\{\frac{d^2}{P_1}I_6 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4 + \sigma_r\left[\frac{2}{\varepsilon}I_1 + \frac{1}{P_1}(1+F)I_1\right]\right\}$$
$$\le 0, \tag{38}$$

since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 is $4\pi^2$.

Now, let $\sigma_r \ge 0$, we necessarily have from inequality (38) that

$$\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}. \tag{39}$$

Hence, if

$$\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \le \frac{4\pi^2}{P_1}, \tag{40}$$

then $\sigma_r < 0$. Therefore, the system is stable.

Therefore, under condition (40), the system is stable and under condition (39) the system becomes unstable.

Theorem 4: $Ep_1 > E'q$ and $\frac{Ep_1}{P} > b\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)$, are the sufficient conditions for the non-existence of overstability.

Proof: For overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it is suffice to find conditions for which equation (26) will admit of solutions with σ_1 real.

Equating real and imaginary parts of equation (26) and eliminating R_1between them, we obtain

$$A_2c_1^2 + A_1c_1 + A_0 = 0\,, \tag{41}$$

where we have put $1 + x = b\,, c_1 = \sigma_1^2$,

and

$$A_2 = E'^2q^2b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)^2\left[\frac{Ep_1}{P} + b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right],$$

$$A_1 = b^3\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)^2\left[\frac{Ep_1}{P} + b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right] + \frac{q^2E'^2b}{P^2}\left[\frac{Ep_1}{P} + b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right]$$
$$+ S_1(b-1)b(Ep_1 - E'q)\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)^2 + T_1q^2E'^2\left[\frac{Ep_1}{P} - b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right],$$

$$A_0 = \frac{b^3}{P^2}\left[\frac{Ep_1}{P} + b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right] + S_1(b-1)\frac{b}{P^2}(Ep_1 - E'q) + T_1b^2\left[\frac{Ep_1}{P} - b\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)\right].$$

Since σ_1 is real for overstability, both the values of $c_1(=\sigma_1^2)$ are positive. Equation (41) implies that this is clearly impossible if

$$Ep_1 > E'q \text{ and } \frac{Ep_1}{P} > b\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right), \qquad (42)$$

for then A_2 , A_1 , A_0 are all positive and equation (41) does not involve any change of sign.

Thus $\boldsymbol{Ep_1 > E'q \text{ and } \frac{Ep_1}{P} > b\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)}$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

6. Conclusions

The double-diffusive convection of a compressible Rivlin-Ericksen viscoelastic fluid in porous medium in the presence of uniform rotation is considered in the present paper. The investigated is motivated by its interesting complexities as a double-diffusion phenomena as well as its direct relevance to astrophysics and geophysics. The conditions under which convective motion in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The main conclusions from the analysis of this paper are as follows:

- For the case of stationary convection, the stable solute gradient and rotation are found to postpone the onset of convection.
- It is also observed for the case of stationary convection that in the presence of rotation, when
$$\frac{1}{P^2}\frac{(1+x)^2}{x} < or > \frac{T_1(1+x)}{x}$$
the medium permeability hastens or postpones the onset of convection which makes the system stable or unstable.
- The system is stable for $G < 1$.
- Both the steady gradient of solute concentration and the presence of rotation are responsible for the introduction of oscillatory modes into the system; these modes were not present when neither of these factors was present.
- The system is stable for $\frac{1}{G-1}\frac{c_p \alpha \kappa}{\nu} \leq \frac{4\pi^2}{P_1}$ and under the condition $\frac{1}{G-1}\frac{c_p \alpha \kappa}{\nu} > \frac{4\pi^2}{P_1}$, the system becomes unstable.
- $Ep_1 > E'q \text{ and } \frac{Ep_1}{P} > b\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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