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THE *p*-GROUP of the STRUCTURE: $(D_26 \times C_2n)$ FOR $n \geq 6$

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Abstract

Efforts are carefully being intensified to calculate , in this paper, the explicit formulae for the number of distinct fuzzy subgroups of the carte- sian product of the dihedral group of order 2^6 with a cyclic group of order of an m power of two for, which $n \geq 6$.

Keywords: Finite *p*-Groups, Nilpotent Group, Fuzzy subgroups, Dihedral Group, Inclusion-Exclusion Principle, Maximal subgroups.

1. Introduction

Since inception, the study of pure mathematics has been extended to some other important classes of finite abelian and nonabelian groups such as the dihedral, quaternion, semidihedral, and hamiltonian groups. Other different approaches have been so far, applied for the classification. The Fuzzy sets were introduced by Zadeh in 1965. Even though, the story of Fuzzy logic started much earlier, it was specially designed mathematically to represent uncertainty and vagueness. It was also, to provide formalized tools for dealing with the imprecision intrinsic to many problems. The term fuzzy logic is generic as it can be used to describe the likes of fuzzy arithmetic, fuzzy mathematical programming, fuzzy topology, fuzzy graph theory and fuzzy data analysis which are customarily called fuzzy set theory. This theory of fuzzy sets has a wide range of applications, one of which is that of fuzzy groups developed by Rosenfield in 1971. This by far, plays a pioneering role for the study of fuzzy algebraic structures. Other notions have been developed based on this theory. These, amongst others, include the notion of level subgroups by P.S. Das used to characterize fuzzy subgroups of finite groups and that of equivalence of fuzzy subgroups introduced by Murali and Makamba which we use in this work. (Please, see [1-9]) Essentially, this work is actually one of the follow up of our paper. (please, see [10]).

By the way, A group is nilpotent if it has a normal series of a finite length *n*.

$G = G_0 \geq G_1 \geq G_2 \geq \cdot \cdot \cdot \geq G_n = \{e\}$,

Where:

$G = G_0 \geq G_1 \geq G_2 \geq \cdots \geq G_n = \{e\}$,

$G_i/G_{i+1} \leq Z(G/G_{i+1})$.

By this notion, every finite p-group is nilpotent. The nilpotence property is a hereditary one. Thus,

1. Any finite product of nilpotent group is nilpotent.
2. If G is nilpotent of a class c, then, every subgroup and quotient group of G is nilpotent and of class $\leq c$.

The problem of classifying the fuzzy subgroups of a finite group has so far experienced a very rapid progress. One particular case or the other have been treated by several papers such as the finite abelian as well as the non-abelian groups. The number of distinct fuzzy subgroups of a finite cyclic group of square-free order has been determined. More- over, a recurrence relation is indicated which can successfully be used to count the number of distinct fuzzy subgroups for two classes of finite abelian groups. They are the arbitrary finite cyclic groups and finite elementary abelian p-groups. For the first class, the explicit formula obtained gave rise to an expression of a well-known central Delannoy numbers. Some forms of propositions for classifying fuzzy subgroups for a class of finite p-groups have been made by Marius Tarnauceaus. It was from there, the study was extended to some important classes of finite non-abelian groups such as the dihedral and hamiltonian groups. And thus, a method of determining the number and nature of fuzzy sub- groups was developed with respect to the equivalence relation. There are other different approaches for the classification. The correspond- ing equivalence classes of fuzzy subgroups are closely connected to the chains of subgroups, and an essential role in solving counting problem is again played by the inclusion - exclusion principle. This hereby leads to some recurrence relations, whose solutions have been easily found. For the purpose of using the Inclusion - Exclusion principle for generating the number of fuzzy subgroups, the finite p-groups has to be explored up to the maximal subgroups. The responsibility of describing the fuzzy subgroup structure of the finite nilpotent groups is the desired objective of this work. Suppose that $(G, , e)$ is a group with identity e. Let $S(G)$ denote the collection of all fuzzy subsets of G. An element λ $S(G)$ is called a fuzzy subgroup of G whenever it satisfies some certain given conditions . Such conditions are as follows :

(i) $\lambda(ab) \geq\in \{\lambda(a), \lambda(b)\}, \quad \forall\, a, b \in G$;

(ii) $\lambda(a^{-1} \geq \lambda(a)$ for any $a \in G$.

And, since $(a^{-1})^{-1} = a$, we have that $\lambda(a^{-1}) = \lambda(a)$, for any a G. Also, by this notation and definition, $\lambda(e) = \sup \lambda(G)$. [Marius [6].

Theorem :The set $FL(G)$ possessing all fuzzy subgroups of G forms a lattice under the usual ordering of fuzzy set inclusion. This is called the fuzzy subgroup lattice of G.

We define the level subset: $\in$

$\lambda G_\beta = \{a \in G/\lambda(a) \geq \beta\}$ for each $\beta \in [0, 1]$

The fuzzy subgroups of a finite p-group G are thus, characterized, based on these subsets. In the sequel, λ is a fuzzy subgroup of G if and only if its level subsets are subgroups in G. This theorem gives a link between $FL(G)$ and $L(G)$, the classical subgroup lattice of G.F

Moreover, some natural relations on $S(G)$ can also be used in the process of classifying the fuzzy subgroups of a finite q-group G. One of

Them is defined by: λ γ iff $(\lambda(a) > \lambda(b)$ $v(a) > v(b)$, a, b $G)$. Alos, two fuzzy subgroups λ, γ of G and said to be distinct if λ v.

As a result of this development, let G be a finite p-group and suppose that $\lambda : G \longrightarrow [0, 1]$ is a fuzzy subgroup of G. Put $\lambda(G) = \{\beta_1, \beta_2, \ldots, \beta_k\}$ with the assumption that $\beta_1 < \beta_2 >$

$> \beta_k$. Then, ends in G is deter- mined by λ.

$\lambda G_{\beta 1} \subset \lambda G_{\beta 2} \subset \cdots \subset \lambda G_{\beta k} = G \quad (a)$

Also, we have that:

$\lambda(a) = \beta_t \Longleftrightarrow t = \max\{r/a \in \lambda G_{\beta r}\} \Longleftrightarrow a \in \lambda G_{\beta t} \backslash \lambda G_{\beta t-1}$,

For any $a \in G$ and $t = 1, \ldots, k$, where by convention, set $\lambda G_{\beta 0} = \varphi$.

2. Methodology

We are going to adopt a method that will be used in counting the chains of fuzzy subgroups of an arbitrary finite p-group G is described. Suppose that $M_1, M_2, \ldots, M_t$ are the maximal subgroups of G, and denote by $h(G)$ the number of chains of subgroups of G which ends in G. By simply applying the technique of computing $h(G)$, using the application of the Inclusion-Exclusion Principle, we have that:

$$h(G) = 2\left(\sum_{r=1}^{t} h(M_r) - \sum_{1 \leq r_1 < r_2 \leq t} h(M_{r_1} \cap M_{r_2}) + \cdots + (-1)^{t-1} h\left(\bigcap_{r=1}^{t} M_r\right)\right) \quad (\#)$$

In [6], (#) was used to obtain the explicit formulas for some positive integers n.

Theorem [1] [Marius]: The number of distinct fuzzy subgroups of a finite p-group of order p^n which have a cyclic maximal subgroup is:

(i) $h(Z_{p^n}) = 2^n$, (ii) $h(Z_p \times Z_{p^{n-1}}) = 2^{n-1}[2 + (n-1)p]$

3. The District Number of The Fuzzy Subgroups of The Nilpotent Group of $(D_{2^3} \times C_{2^m})$ For $m \geq 3$

Proposition 1 (see [11]) : Suppose that $G = Z_4 \times Z_{2^n}$, $n \geq 2$. Then,

$h(G) = 2^n[n^2 + 5n - 2]$

Proof : G has three maximal subgroups of which two are isomorphic to $Z_2 \times Z_{2^n}$ and the third is isomorphic to $Z_4 \times Z_{2^{n-1}}$.

Hence, $h(Z_4 \times Z_{2^n}) = 2h(Z_2 \times Z_{2^n}) + 2^1 h(Z_2 \times Z_{2^{n-1}}) + 2^2 h(Z_2 \times Z_{2^{n-2}})$
$+ 2^3 h(Z_2 \times Z_{2^{n-3}}) + 2^4 h(Z_2 \times Z_{2^{n-4}}) + \cdots + 2^{n-2} h(Z_2 \times Z_{2^2})$

$$= 2^{n+1}[2(n+1) + \sum_{j=1}^{n-2} [(n+1) - j]$$

$= 2^{n+1}[2(n+1) + \frac{1}{2}(n-2)(n+3)] = 2^n[n^2 + 5n - 2], n \geq 2$

We have that: $h(Z_4 \times Z_{2^{n-1}}) = 2^{n-1}[(n-1)^2 + 5(n-1) - 2]$

$= 2^{n-1}[n^2 + 3n - 6], n > 2$ Q

Corrolary1: Following the last proposition, $h(Z_4 \times Z_{2^5})$, $h(Z_4 \times Z_{2^6})$, $h(Z_4 \times Z_{2^7})$ and $h(Z_4 \times Z_{2^8})$ = 1536, 4096, 10496 and 26112 respectively.

Theorem A (see [12]) : Let $G = D_{2^n}$ C_2, the nilpotent group formed by the cartesian product of the dihedral group of order 2^n and a cyclic group of order 2. Then, the number of distinct fuzzy subgroups of G is given by: $h(G) = 2^{2n}(2n+1) - 2^{n+1}, n > 3$

Proof:

The group $D_{2^n} \times C_2$, has one maximal subgroup which is isomorphic to Z_2 $Z_{2^{n-2}}$, two maximal subgroups which are isomorphic to $D_{2^{n-1}}$ C_2, and 2^2 which are isomorphic to D_{2^n}.

It thus, follows from the Inclusion-Exclusion Principle using equation,

$\frac{1}{2}h(D_{2^n} \times C_2) = h(Z_2 \times Z_{2^{n-1}}) + 4h(D_{2^n}) - 8h(D_{2^{n-1}}) - 2h(Z_2 \times Z_{2^{n-2}}) + 2h(D_{2^{n-1}} \times C_2)$

By recurrence relation principle we have:

$\frac{1}{2}h(D_{2^n} \times C_2) = h(Z_2 \times Z_{2^{n-1}}) + 4h(D_{2^n}) - 8h(D_{2^{n-1}}) - 2h(Z_2 \times Z_{2^{n-2}}) + 2h(D_{2^{n-1}} \times C_2)$

By recurrence relation principle we have :

$h(D_{2^n} \times C_2) = 2^{2n}(2n+1) - 2^{n+1}, n > 3$

By the fundermental principle of mathematical induction,

set F(n) = $h(D_{2^n} \times C_2)$, assuming the truth of F(k) $= h(D_{2^k} \times C_2) = 2h(Z_2 \times Z_{k-1})$
$+ 8h(D_{2^k} - 16hD_{2^{k-1}} - 4h(Z_2 \times Z_{k-2}) + 4h(D_{2^{k-1}} \times C_2) = 2^{2k}(2k+1) - 2^{k+1}$,
F(k+1) $= h(D_{2^{k+1}} \times C_2) = 2h(Z_2 \times Z_{2^k}) + 8h(D_{2^{k+1}} - 16h(D_{2^k} - 4h(Z_2 \times Z_{k-1})$
$+ 4h(D_{2^k} \times C_2) = 2^2[2^{2k}(2k-3) - 2^k]$, which is true. Q

Proposition 2 (see [2]): Suppose that $G = D_{2^n} \times C_4$. Then, the number of distinct fuzzy subgroups of G is given by:

$$2^{2(n-2)}(64n + 173) + 3\sum_{j=1}^{n-3} 2^{(n-1+j)}(2n+1-2j)$$

Proof:

$\frac{1}{2}h(D_{2^n} \times C_4) = h(D_{2^n} \times C_2) + 2h(D_{2^{n-1}} \times C_4) - 4h(D_{2^{n-1}} \times C_2) + h(Z_4 \times Z_{2^{n-1}})$
$- 2h(Z_2 \times Z_{2^{n-1}}) - 2h(Z_4 \times Z_{2^{n-2}}) + 8h(Z_2 \times Z_{2^{n-2}}) + h(Z_{2^{n-1}}) - 4h(Z_{2^{n-2}})$

$h(D_{2^n} \times C_4) = (n-3).2^{2n+2} + 2^{2(n-3)}(1460) + 3[2^n(2n-1) + 2^{n+1}(2n-3) +$

$2^{n+2}(2n-5)+\cdots+7(2^{2(n-2)})]$

$n-3$

$=(n-3).2^{2n+2}+2^{2(n-3)}(1460)+3$ $2^{n-1+j}(2n+1-2j)$

$j=1$ $n-3$

$=2$ $\sum 2(n\ 2)$ $n\ 1+j(64n + 173) + 3$

$2\ -$ $(2n+1-2j)$

$j=1$

Proposition 3 : Let G be an abelian p-group of type $Z_p Z_p \times Z_{p^n} \times$ where p is a prime and n 1. The number of distinct fuzzy subgroups of G is $h(Z_p\ Z_p\ Z_{p^n}) = 2^n p(p+1)(n \times 1)(3+np+2p)+(2^n\ \_\ 2)p^3$ $\_\ 2^{n+1}(n\ \_\ 1)p^3+2^n[p^3+4(1+p+p^2)]$.

Proof: There exist exactly $1+p+p^2$ maximal subgroups for the abeliantype $Z_p \times Z_p \times Z_{p^n}$, [Berkovich(2008)]. One of them is isomorphic to $Z_p\ Z_p\ Z_{p^{n-1}}$, while each of the remaining $p+p^2$ is isomorphic to $Z_p \times Z_{p^n}$.
Thus, by the application of the Inclusion-Exclusion Principle,we have as follows: $h(Z_p \times Z_p \times Z_{p^n}) = 2^n p(p+1)(n\ 1)(3+np+2p)+(2^n\_\ 2)p^3\_\ 2^{n+1}(n\ 1)p^3 + 2^n[p^3+4(1+p+p^2)]$ And thus,

$h(Z_p \times Z_p \times Z_{p^{n-2}}) = 2^{n-2}[4+(3n-5)p+(n^2-5)p^2+(n^2-5n+8)p^3]-2p^2.$

Q

Corrolary 2: From (3) above, obsreve that, we are going to have that:

$h(Z_3 \times Z_3 \times Z_{3^n}\ {}^{n+})\ {}^2 = 2$ $[18n + 9n + 26] - 54$

Similarly, for $p=5$, using the same analogy, we have

$h(Z_5 \times Z_5 \times Z_{5^n}) = 2[30h(Z_5 \times Z_{5^n})+h(Z_5 \times Z_5 \times Z_{5^{n-1}})$

$-p\ h(Z_{5^n})-30h(Z_{5^{n-1}})+125]$ ^{3}

And for $p=7$,

$h(Z_7{\times}Z_7{\times}Z_{7^n}) = 2[56h(Z_7{\times}Z_{7^n})+h(Z_7{\times}Z_7{\times}Z_{7^{n-1}})-343h(Z_{7^n})-56h(Z_{7^{n-1}})+343]$

We have, in general, $h(Z_p \times Z_p \times Z_{p^{n-2}}) = 2^{n-2}[4+(3n-5)p+(n^2-5)p^2+(n^2-5n+8)p^3]-2p^2$ Q

Proposition (please, see [10]):
Let $G=(D_{2^3} \times C_{2^m})$ for $m \geq 3$. Then, $h(G) = m(89-23m)+(85)2^{m+3}-124$

Proof:

There exist seven maximal subgroups, of which one is isomorphic to $D_{2^3} \times C_{2^{m-1}}$, two being isomorphic to $C_{2^m} \times C_2 \times C_2)$, two isomorphic to C_{2^m} C_2, and one each isomorphic to C_{2^m} C_4, and C_{2^m} respectively.
Hence, by the inclusion - exclusion principle, using the propositions [1], [2], [3], and Theorem [1] we have that

$\frac{1}{2}h(G) = h(D_{2^3} \times C_{2^{m-1}})+2h(C_{2^m} \times C_2) \times C_2)+2h(C_{2^m} \times C_2)+2h(C_{2^m} \times C_4)+h(C_{2^m})-12h(C_{2^m} \times C_2)-6h(C_{2^{m-1}} \times C_2) \times C_2)-3h(C_{2^{m-1}}) \times C_4)+28h(C_{2^{m-1}} \times C_2)+2h(C_{2^{m-1}} \times C_2) \times C_2)+4h(C_{2^m} \times C_2)+h(C_{2^{m-1}} \times C_4)-35h(C_{2^{m-1}} \times C_2)-7h(C_{2^{m-1}} \times C_2)+h(C_{2^{m-1}} \times C_2)$

$=h(D_{2^3} \times C_2^{m-1})+2h(C_{2^m} \times C_2) \times C_2)-6h(C_{2^m} \times C_2)+h(C_{2^m} \times C_4)+h(C_{2^m})-4h(C_{2^{m-1}} \times C_2) \times C_2)-2h(C_{2^{m-1}}) \times C_4)+8h(C_{2^{m-1}} \times C_2)$

$=h(D_{2^3} \times C_2^{m-1})+2^{m+2}(6m^2+7m+9)-32-(6)2^m(2m+2)+8m(2^m)-2^{m+2}\ 6m^2 - 5m + 8 + + 2^m(m^2+5m-2)-2^m\ (3m+m^2-6)+2^m = h(D_{2^3} \times$ 2^6

$C_2^{m-1})+2^m(46m-4)+2^m+32 = h(D_{2^3} \times C_2^{m-1})+2^m(46m-3)+32$

Hence , $h(G) = 2h(D_{2^3} \times C^{m-1})+2^{m+1}(46m-3)+64 = 2^{m+1}(46m-3)+64+2[2^m(46m-49)+64+2h(D_{2^3} \times C^{m-2})] = 2^{m+1}(46m-3)+64+2^{m+1}(46m-49)+2^7+2^2h(D_{2^3} \times C_2^{m-2}) = 2^{m+2}(46mm-3)+2^6+2^{m+1}(46m-49)+2^7+2^2[2^{m-1}(46m-95)+64+2h(D_{2^3} \times C_2^{m-3})$

$h(D_{2^3{}_n} \times C_{2^m}) = (46m-3).2^{m+1}+2^6+(46m-49)2^{m+1}+2^7+(46m-95)2^{m+1}+2^8+2^3\ h(D_{2^3} \times C_{2^{m-3}})$

$=2^{m+1}.[(46m-3)+(46m-49)+(46m-95)]+2^6+2^7+2^8+2^3h(D_{2^3} \times C_2^{m-3})$

$=$ $2^6+2^7+2^8+\cdots+2^{5+k}$

$$+ \quad 2^{m+1}.|46mk+ \quad \overbrace{(-3-49-95\cdots}^{\text{series (1)}}\underbrace{(-3-46(k-1)))|}_{\text{series (2)}} \quad x$$

$+2^k h(D_{2^3} \times C_{2^{m-k}}),\ k \in \{1, 2.3. \cdots n \in N\}$

For the series (1) , we have that, $U_m = 2^6.2^{m-1} = 2^{5+k}$, $m + 5 = k + 5, \Rightarrow m = k$. We have that $S_{n=k} = 2^6[\frac{2^k-1}{2-1}] = 2^6(2^k.1)$

And for the second series (2), we have that , $T_m = -3 + (m-1)(-46) = -3 - 46(k-1) \Rightarrow m-1 = k-1, n = k$ Hence , $S_m = k = \frac{k}{2}[2(-3) + (k-1)(-46)] = \frac{k}{2}(-6 - 46k + 46) = \frac{k}{2}(40 - 46k)$. We have that $h(D_{23n} \times C_{2^m}) = \frac{k}{2}(40 - 46k) + 2^6(2^k.1) + 2^k h(D_3 \times C_{2^{m-k}}$. By setting $m = k$ we have that $k = m - 3$.Hence , $h(D_{2^3} \times C_{2^m}) = (m-3)(20-23m) + 2^6(2^{m-3}-1) + 2^m - 3h(D_3 \times C_{2^3})$

$h(G) = (m-3)(20-23m) + 2^6(2^{m-3}-1) + 2^{m-3}(5376) = (m-3)(20-23m) + 2^{m-3} - 2^6 + 2^{m+3}(21) = 20m - 23m^2 - 60 + 69m + 2^{m+3} - 2^6 + (21)2^{m+3} = (89m - 23m^2 - 60) + 2^{m+3} - 2^6 + (21)2^{m+3} = m(89 - 23m) - 124 + (85)2^{m+3}$ Q

Determining the number of distinct fuzzy subgroups for $G = D_26 \times C_2n$, $n \geq 6$.

Suppose that $G = (D_{2^6} \times C_{2^n})$ for $n \geq 6$. Then ,

$\frac{1}{2}h(G) = h(D_{2^6} \times C_{2^{n-1}}) + 2h(D_{2^5} \times C_{2^n}) + 2h(D_{2^n} \times C_{2^4}) + h(D_{2^n} \times C_{2^5})$
$-4h(D_{2^5} \times C_{2^{n-1}}) - 4h(C_{2^4} \times C_{2^n}) - 2h(C_{2^5}) \times C_{2^{n-1}}) + 8h(C_{2^4} \times C_{2^{n-1}}) - 3h(C_{2^n})$

So, $h(G) = 2h(D_{2^6} \times C_{2^{n-1}}) + 4h(D_{2^5} \times C_{2^n}) + 4h(D_{2^n} \times C_{2^4})$

$+ 2h(D_2n \times C_25) - 8h(D_25 \times C_2n-1) - 8h(C_24 \times C_2n) - 4h(C_25) \times C_2n-1)$
$+ 16h(C_24 \times C_2n-1) - 6h(C_2n)$ Q

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Competing of interests statement

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