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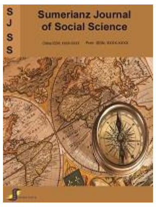

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Population Wealth and Business Cycles with Job Discrimination

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Abstract

This study confirms existence of business cycles in a model of economic growth and population change with discrimination against women proposed. We generalize Zhang's model by allowing constant coefficients to be time-dependent (Zhang, 2017). Zhang's model deals with dynamic interdependence between wealth accumulation and endogenous birth and mortality rates with gender discrimination. The production technology and markets influenced by Solow's neoclassical growth model. The basic mechanisms for population changes are influenced by the Barro-Becker fertility choice model and the Haavelmo population model. The model synthesizes these dynamic forces in a compact framework by applying Zhang's utility function. This study examines properties of the generalized model and identifies business cycles due to exogenous periodic shocks.

Keywords: Business cycles; Periodic shocks; Birth and mortality rates; Propensity to have children; Gender difference in time distribution; Population growth; Wealth accumulation.

1. Introduction

This study confirms existence of business cycles in a model of economic growth and population change with discrimination against women proposed. We generalized Zhang's model by allowing constant coefficients to be time-dependent (Zhang, 2017). Zhang's model deals with dynamic interdependence between wealth accumulation and endogenous birth and mortality rates with gender discrimination. The production technology and markets influenced by Solow's neoclassical growth model (Solow, 1956). The basic mechanisms for population changes are influenced by the Barro-Becker fertility choice model and the Haavelmo population model. Population change rate is birth rate minus and mortality rate. There are many studies on economic growth and birth rates (Adsera, 2005; Barro and Becker, 1989; Becker *et al.*, 1990; Chu *et al.*, 2012; Galor and Weil, 1996; Hock and Weil, 2012). There are also studies on quality-quantity trade-offs on children (Bosi and Seegmuller, 2012; Doepke, 2004; Galor and Weil, 1999). Varvarigos and Zakaria (2013), study interdependence between fertility choice and expenditures on health (see also, Bhattacharya and Qiao (2007); Manuelli and Seshadri (2009). There are also many formal models on economic growth and mortality rates (e.g., Strulik (2008); Galor (2012); Kirk (1996); Ehrlich and Lui (1997); Acemoglu and Johnson (2007). Zhang (2017), builds a model by synthesizing various dynamic forces in a compact framework by applying Zhang's utility function. This study extends Zhang's model and identifies business cycles due to exogenous periodic shocks. Although there are many studies on business cycles (Chiarella and Flaschel, 2000; Gandolfo, 2005; Lorenz, 1993; Puu, 2011; Shone, 2002; Stachurski *et al.*, 2014; Zhang, 1991;2005;2006), there are a few theoretical models of endogenous population which are applied to show business cycles. This study makes a contribution to the literature of business cycles by identifying business cycles in a formal growth model with endogenous population. The rest of the paper is organized as follows. In Section 2, we generalize Zhang's model by allowing constant coefficients to be time-dependent. In Section 3, we generalize Zhang's model and simulate the model when all coefficients are constant. In Section 4, we identify existence of business cycles due to various exogenous periodic oscillations. In Section 5, we conclude the study.

2. The Basic Model

As the model in this study is to allow constant coefficients in Zhang's model to be time-dependent, we refer to Zhang (2017) for explaining modelling. Markets are perfectly competitive. There is only one sector model as in the Solow model Solow (1956). The sector produces a single commodity for investment and consumption. Depreciation rate of capital is denoted by $\delta_k(t)$. Input factors are fully utilized. Saving is made by households. The population is composed of male and female populations. A family is composed of consists of husband, wife and children. All the families are identical. Man and woman are denoted with subscript indexes $q = 1$ and $q = 2$, respectively. We use $N(t)$ to stand for the adult population of each gender. Let $T_q(t)$ and $\bar{T}_q(t)$ stand for work time and time spent on taking care of children of gender q. We use $N_q(t)$ to represent the qualified labor force of gender q. We have:

$$N_q(t) = h_q(t)\, T_q(t)\, N(t), \bar{N}(t) = N_1(t) + N_1(t), (1)$$

where $h_q(t)$ is gender $q's$ level of human capital and $\bar{N}(t)$ is total labor supply employed in time t for production.

2.1. The Production Sector

We use $K(t)$ and $F(t)$ to stand for capital input and output level. Production function is specified as

$$F(t) = A(t)\, K^{\alpha(t)}(t)\, \bar{N}^{\beta(t)}(t), \alpha(t), \beta(t) > 0, \alpha(t) + \beta(t) = 1, (2)$$

where $\alpha(t)$ and $\beta(t)$ are respectively the output elasticities of capital and qualified labor input and $A(t)$ is the total factor productivity. Labor and capital are paid their marginal products. Firms earn zero profits. We use $w(t)$ to represent the wage rate of per unit of qualified work time in fair labor market where workers earn their marginal value of labor. This study considers existence of gender discrimination in labor market (e.g., Dozier *et al.* (2013); Heyman *et al.* (2013); Jonathan and Kerwin (2013); Lanning (2014); Zhang (2014). There is a fraction $\phi(t)$ of women's fair share of the gender's labor taken away by firms from women. The rate $\phi(t)$ is called the discrimination rate against woman. The total cost of the female labor force due to discrimination is $\phi(t) h_2(t) T_2(t) N(t)$. The production sector's profit is given as follows

$$F(t) - \left(r(t) + \delta_k(t)\right)K(t) - w(t)\, h_1(t)\, T_1(t)\, N(t) - \left(1 - \phi(t)\right)w(t)\, h_2(t)\, T_2(t)\, N(t),$$

Profit maximization yield the following marginal conditions

$$r(t) + \delta_k(t) = \frac{\alpha(t)\, F(t)}{K(t)}, w_1(t) = \frac{\beta(t)\, h_1(t)\, F(t)}{\bar{N}(t)}, w_2(t) = \frac{\beta(t)\, h_2(t)\, F(t)}{\bar{N}(t)}, (3)$$

where $w_q(t)$ is the wage rate of per unit of work time by gender q:

$$w_1(t) \equiv w(t)\, h_1(t), w_2(t) \equiv \left(1 - \phi(t)\right)w(t)\, h_2(t).$$

2.2. The Current and Disposable Incomes

The family decides five variables: leisure time, work time, consumption level of commodity, number of children, and saving. We use $\bar{k}(t)$ to represent wealth per family. We have $\bar{k}(t) = K(t)/N(t)$. The family's current income $y(t)$ from interest income and wage payments is

$$y(t) = r(t)\, \bar{k}(t) + w_1(t)\, T_1(t) + w_2(t)\, T_2(t).$$

The family's disposable income is the sum of the current income and the value of wealth:

$$\hat{y}(t) = y(t) + \bar{k}(t). (4)$$

2.3. The Cost of Children Caring

Let $n(t)$ and$p_b(t)$ stand for birth rate and cost of birth respectively. Parents spend time and the following cost on their children (see also, (Becker, 1981); and Barro and Becker (1989)

$$p_b(t) = n(t)\, \bar{k}(t). (5)$$

The relation between fertility rate and the parent's time on raising children is specified as follows:

$$\bar{T}_q(t) = \theta_q(t)\, n(t), \theta_q(t) \geq 0. (6)$$

2.4. The Budget and Time Constraint

The disposable income is used up for saving $s(t)$, consumption of goods $c(t)$, and bearing children $p_b(t)$. The budget constraint implies

$$p(t)c(t) + s(t) + \bar{k}(t)n(t) = \hat{y}(t). (7)$$

The leisure time of gender q is presented by $\tilde{T}_q(t)$. The available time T_0 is used for work, children caring and leisure. The time constraint for each worker is

$$T_q(t) + \bar{T}_q(t) + \tilde{T}_q(t) = T_0. (8)$$

Substitute (8) into (7)

$$p(t)\, c(t) + s(t) + \bar{k}(t)\, n(t) + \bar{T}_1(t)\, w_1(t) + \bar{T}_2(t)\, w_2(t) + \tilde{T}_1(t)\, w_1(t) + \tilde{T}_2(t)\, w_2(t) = \bar{y}(t), (9)$$

in which

$$\bar{y}(t) \equiv \left(1 + r(t)\right)\bar{k}(t) + \left(w_1(t) + w_2(t)\right)T_0.$$

From (6) and (9), we have

$$c(t) + s(t) + \widetilde{w}(t)\, n(t) + \tilde{T}_1(t)\, w_1(t) + \tilde{T}_2(t)\, w_2(t) = \bar{y}(t), (10)$$

where

$$\widetilde{w}(t) \equiv \bar{k}(t) + h(t)\, w(t), h(t) \equiv \theta_1(t)\, h_1(t) + \left(1 - \phi(t)\right)\theta_2\, h_2(t).$$

2.5. The Utility and Optimal Behavior

Following Zhang (2015); Zhang (2017), the utility is a function of $c(t)$, $s(t)$, $\tilde{T}_q(t)$, and $n(t)$ as follows:

$$U(t) = c^{\xi_0(t)}(t)\, s^{\lambda_0(t)}(t)\, \tilde{T}_1^{\sigma_{01}(t)}(t)\, \tilde{T}_2^{\sigma_{02}(t)}(t)\, n^{\upsilon_0(t)}(t),$$

where $\xi_0(t)$ is the propensity to consume, $\sigma_{0q}(t)$ the gender $q's$ propensity to use leisure time, $\lambda_0(t)$ the propensity to have wealth, and $\upsilon_0(t)$ the propensity to have children. Marginal conditions for maximizing the utility subject to (10) implies:

$$c(t) = \xi(t)\, \bar{y}(t), s(t) = \lambda(t)\, \bar{y}(t), \tilde{T}_q(t) = \frac{\sigma_q(t)\, \bar{y}(t)}{w_q(t)}, n(t) = \frac{\upsilon(t)\, \bar{y}(t)}{\widetilde{w}(t)}, (11)$$

where

$$\xi(t) \equiv \rho(t)\,\xi_0(t), \lambda(t) \equiv \rho(t)\,\lambda_0(t), \sigma_q(t) \equiv \rho(t)\,\sigma_{q0}(t),$$
$$v(t) \equiv \rho(t)\,v_0(t), \rho(t) \equiv \frac{1}{\xi_0(t) + \lambda_0(t) + \sigma_{10}(t) + \sigma_{20}(t) + v_0(t)}.$$

2.6. Population Change

Let $n(t)$ and $d(t)$ represent birth rate and mortality rate, respectively. The population change rate is birth rate minus mortality rate

$$\dot{N}(t) = \big(n(t) - d(t)\big)N(t), (12)$$

The birth rate is determined by (11). Being influenced by the literature of economic growth with endogenous population (e.g., Haavelmo (1954); Razin and Ben-Zion (1975); Stutzer (1980); Yip and Zhang (1997); Zhang (2017), we determine mortality rate as follows:

$$d(t) = \frac{\bar{v}(t)\,N^{b(t)}(t)}{\bar{y}^{a(t)}(t)}, (13)$$

in which $\bar{v}(t) \geq 0$, $a(t) \geq 0$. Substituting (10) and (13) into (12) yields:

$$\dot{N}(t) = \left(\frac{v(t)\,\bar{y}(t)}{\widetilde{w}(t)} - \frac{\bar{v}(t)\,N^{b(t)}(t)}{\bar{y}^{a(t)}(t)}\right)N(t). (14)$$

2.7. Wealth Dynamics

Change in the family's wealth is savings minus dissavings. We thus have

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda(t)\bar{y}(t) - \bar{k}(t). (15)$$

2.8. Demand and Supply of Goods

Output is used for net savings and depreciation of capital stock. We have

$$S(t) + C(t) - K(t) + \delta_k(t)K(t) = F(t), (16)$$

where $S(t) - K(t) + \delta_k(t)K(t)$ is the sum of the net saving and depreciation and

$$S(t) \equiv s(t)\,N(t), C(t) \equiv c(t)\,N(t), K(t) \equiv \bar{k}(t)\,N(t).$$

We built the model.

3. Analyzing Dynamic Properties of the Model

This section studies dynamics of the model. We define a new variable

$$z(t) \equiv \frac{r(t) + \delta_k(t)}{w(t)}.$$

The following lemma implies that the motion of the national economy is determined by two differential equations.

3.1. Lemma

We determine the movement of the economy with two differential equations:

$$\dot{z}(t) = \tilde{\Omega}_z(z(t), N(t), t),$$
$$\dot{N}(t) = \tilde{\Omega}_N(z(t), N(t), t), (17)$$

in which $\tilde{\Omega}_z$ and $\tilde{\Omega}_N$ are functions of $z(t)$, $N(t)$ and t given in the Appendix. Other variables are determined as functions of $z(t)$, $N(t)$ and t as follows: $\bar{k}(t)$ by (A11) $\rightarrow$ $r(t)$ and $w_q(t)$ by (A2) $\rightarrow$ $\bar{N}(t)$ by (A16) $\rightarrow$ $\bar{y}(t)$ by (A3) $\rightarrow$ $c(t)$, $s(t)$, $\tilde{T}_q(t)$, and $n(t)$ by (11) $\rightarrow$ $\bar{T}_q(t)$ by (6) $\rightarrow$ $T_q(t)$ by (A4) $\rightarrow$ $K(t)$ by (A1) $\rightarrow$ $F(t)$ by (2).

We simulate the model. In the rest of this section, we summarize the simulation results obtained by Zhang (2017) for a special case of our model when all exogenous time dependent coefficients are constant. We choose $\delta_k = 0.05$ and $T_0 = 24$. The other parameters take on the following values

$$\alpha = 0.34, \lambda_0 = 0.6, \xi_0 = 0.2, v_0 = 0.4, \sigma_{10} = 0.15, \sigma_{10} = 0.15, A = 1,$$
$$a = 0.4, b = 0.5, h_1 = 3, h_2 = 2.6, \theta_1 = 2, \theta_2 = 5, \bar{v} = 1, \phi = 0.2. (18)$$

The discrimination rate is 0.2. Initial conditions are

$$z(0) = 1.6, N(0) = 21.$$

Figure 1 shows the simulation result. The equilibrium values of the variables are listed as follows

$$N = 23.73, K = 446.49, \bar{N} = 1260.3, F = 885.6, n = d = 0.59, r = 0.62,$$
$$w_1 = 1.86, w_2 = 1.11, \widetilde{w} = 89, \bar{k} = 78.4, T_1 = 11.6, T_2 = 2.3, \tilde{T}_1 = 12.3, \tilde{T}_2 = 18.8,$$
$$\bar{T}_1 = 1.17, \bar{T}_2 = 2.93, c = 26.1.$$

The system's two eigenvalues are: -0.393 and -0.369. As the two eigenvalues are negative, we can conduct comparative dynamic analysis effectively.

4. Comparative Dynamic Analysis

This section simulates the model when coefficients are time dependent. This section is especially concerned with existence of business cycles due to different time-dependent exogenous shocks. We introduce symbol $\bar{\Delta}x(t)$ to represent the change rate of variable $x(t)$ in percentage due to changes in the parameter values.

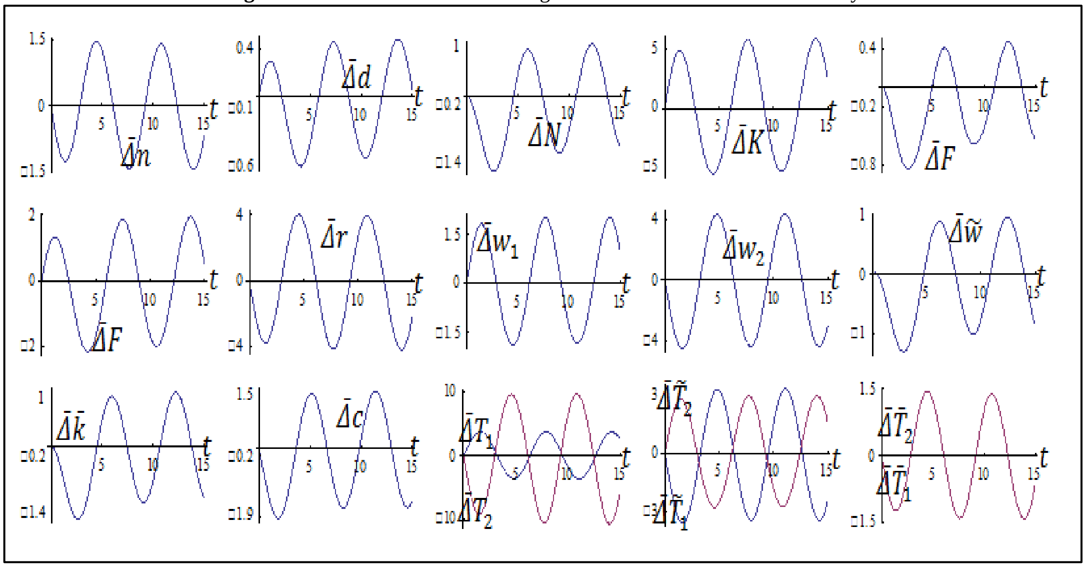
4.1. The Discrimination Rate Against Woman Periodically Oscillates

There are various ways of discrimination against women. Boserup (1970), argues for existence of a curvilinear relationship between economic growth and status of women. There is a widening gap between men and women in initial stages of economic growth. To show impact of discrimination against woman, we now simulate a case that discrimination rate against women periodically oscillates as follows:

$$\phi(t) = 0.2 + 0.05\, sin(t).$$

Figure 1 plots the simulation results. We see that periodic oscillations in the discrimination leads to business cycles in our model.

Figure-1. The Discrimination Rate against Woman Oscillates Periodically



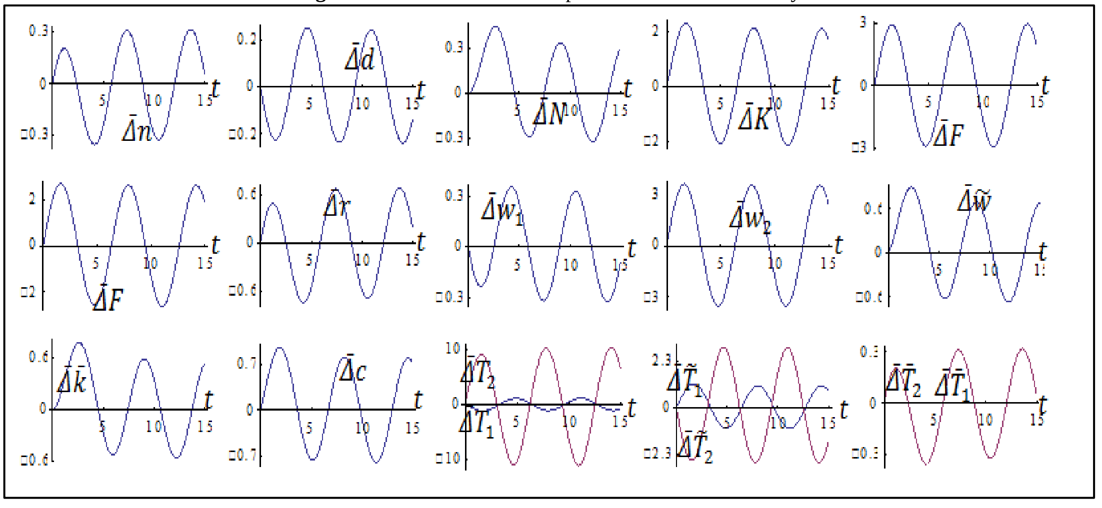
4.2. Woman's Human Capital Oscillates Periodically

Traditional neoclassical economic theory holds that gender inequalities associated with human capital gap will wither away as an economy experiences high economic growth (e.g., Beneria and Feldman (1992); Truong (1997); Forsythe *et al.* (2000); Dolado *et al.* (2001); Duflo (2012). Stotsky (2006) points out: "the neoclassical approach examines the simultaneous interaction of economic development and the reduction of gender inequalities. It sees the process of economic development leading to the reduction of these inequalities and also inequalities hindering economic development." We now allow the mother's human capital to oscillate periodically as follows:

$$h_2(t) = 2.6 + 0.1\, sin(t).$$

Figure 2 plots the simulation results. Birth rates oscillate as the female population's human capital oscillates. It should be noted that some researches find positive interdependence between life expectancy and the aggregate human capital level (e.g., Blackburn and Cipriani (2002); Boucekkine *et al.* (2002). Our result also demonstrates the same trend if we consider the mortality rate oscillate in association with to the life expectancy.

Figure-2. Woman's Human Capital Oscillates Periodically



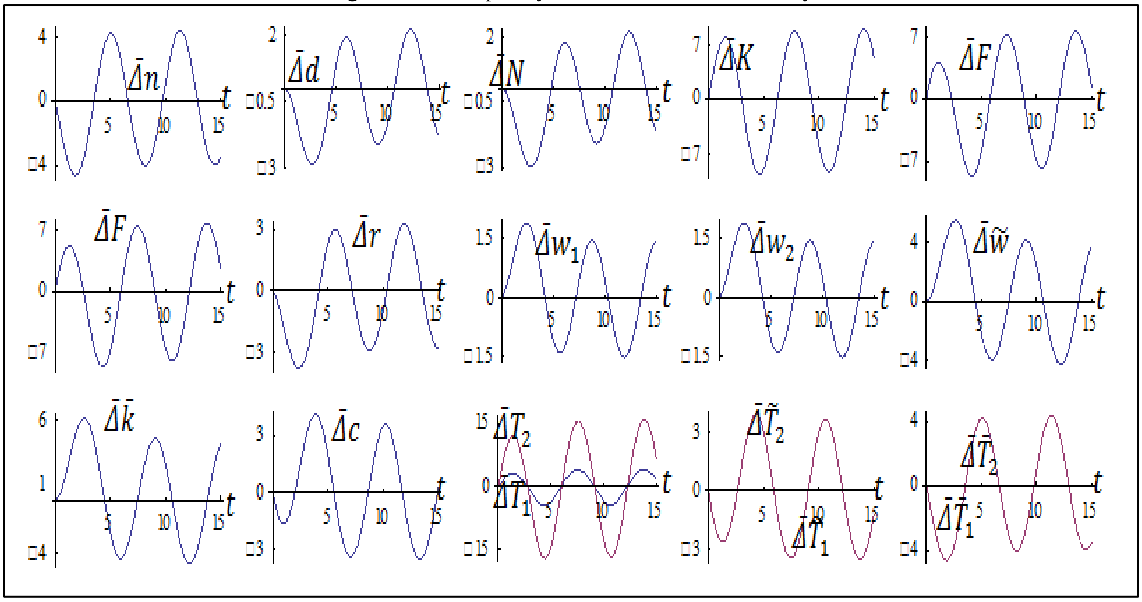
4.3. The Propensity to Save Oscillates Periodically

We allow the propensity to save oscillate periodically as follows:

$$\lambda_0(t) = 0.6 + 0.01\, sin(t).$$

Figure 3 plots the simulation results.

Figure-3. The Propensity to Save Oscillates Periodically



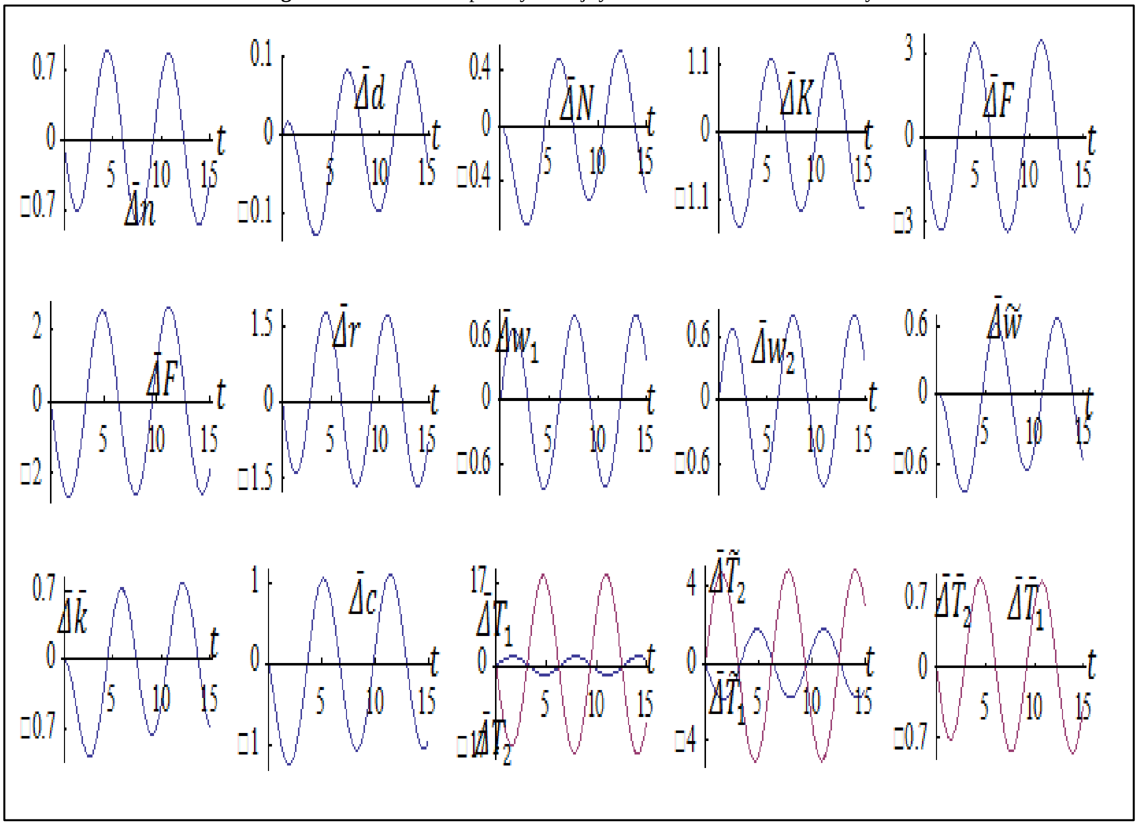
4.4. Woman's Propensity to Enjoy Leisure Oscillates Periodically

Woman may have different preferences in different stages of economic development. We now study the following periodic oscillations in women's propensity to enjoy leisure:

$$\sigma_{02}(t) = 0.15 + 0.01\, sin(t).$$

Figure 4 plots the simulation results.

Figure-4. Woman's Propensity to Enjoy Leisure Oscillates Periodically



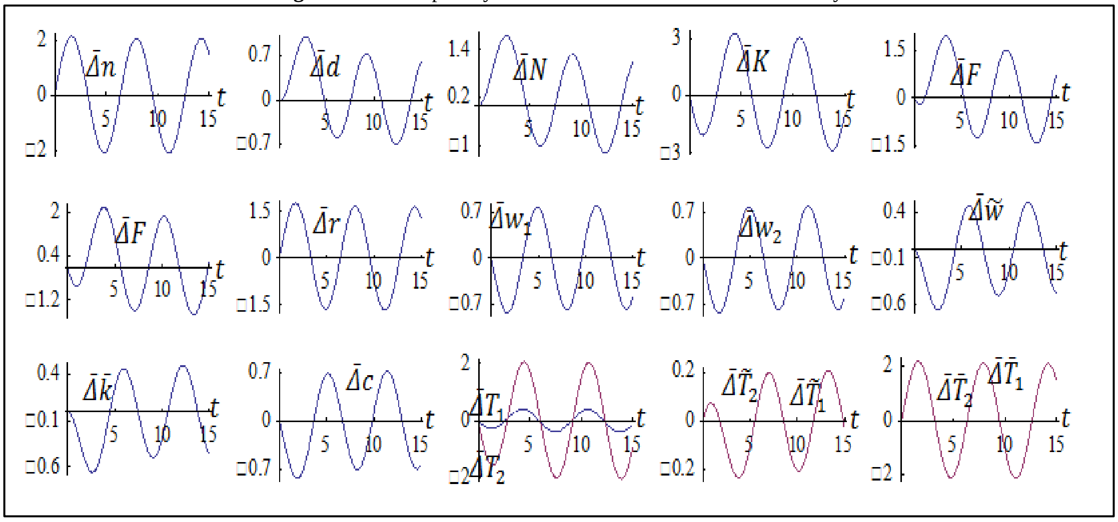
4.5. The Propensity to have Children Oscillates Periodically

We now study the case that the propensity to have children oscillate periodically as follows:

$$\upsilon_0(t) = 0.4 + 0.02\, sin(t).$$

Figure 5 plots the simulation results.

Figure-5. The Propensity to Have Children Oscillates Periodically



5. Concluding Remarks

This study confirmed existence of business cycles in a model of economic growth and population change with discrimination against women proposed. We generalized Zhang's model by allowing constant coefficients to be time-dependent (Zhang, 2017). Zhang's model deals with dynamic interdependence between wealth accumulation and endogenous birth and mortality rates with gender discrimination. The production technology and markets influenced by Solow's neoclassical growth model. The basic mechanisms for population changes are influenced by the Barro-Becker fertility choice model and the Haavelmo population model. The model synthesizes these dynamic forces in a compact framework by applying Zhang's utility function. This study examined properties of the generalized model and identified business cycles due to exogenous periodic shocks. As the model is influenced by a few well-known models in the literature of economic theory and each model generates a vast literature, we may generalize and extend the model. We may generalize our model by using more general utility or production functions.

Appendix: Checking the Lemma

From (3), we get

$$z \equiv \frac{r+\delta_k}{w} = \frac{\tilde{\alpha}\,\bar{N}}{K}, (A1)$$

in which $\tilde{\alpha} \equiv \alpha/\beta$. Insert (A1) in (2) and (3)

$$r = \alpha A \left(\frac{z}{\tilde{\alpha}}\right)^{\beta} - \delta_k, w = \beta A \left(\frac{\tilde{\alpha}}{z}\right)^{\alpha}, w_1 = w\, h_1, w_2 = (1-\phi) w\, h_2. (A2)$$

From the definition of $\bar{y}$ and (3), we get

$$\bar{y} = (1+r)\bar{k} + h_0 w, (A3)$$

in which $h_0 \equiv (h_1 + (1-\phi)h_2)T_0$. From (8) and (11), we have

$$T_q = T_0 - \bar{T}_q - \tilde{T}_q = T_0 - \left(\frac{\theta_q \upsilon}{\tilde{w}} + \frac{\sigma_q}{w_q}\right)\bar{y}. (A4)$$

Substitute (A3) into (A4)

$$T_q = \chi_q - \frac{\tilde{r}_q \bar{k} + \bar{r}_q}{\tilde{w}} - r_q \bar{k}, (A5)$$

in which

$$\chi_q = T_0 - \frac{h_0\, w\, \sigma_q}{w_q}, \tilde{r}_q \equiv \theta_q\, \upsilon(1+r), \bar{r}_q \equiv h_0\, \theta_q\, \upsilon\, w, r_q \equiv \frac{(1+r)\sigma_q}{w_q}.$$

Insert (A5) in (1)

$$\frac{\bar{N}}{N} = h_1\, T_1 + h_2\, T_2 = \chi - \frac{\tilde{r}\,\bar{k} + \bar{h}_0}{\tilde{w}} - \tilde{r}_0\, \bar{k}, (A6)$$

in which

$$\chi \equiv h_1\, \chi_1 + h_2\, \chi_2, \tilde{r} \equiv h_1\, \tilde{r}_1 + h_2\, \tilde{r}_2, \bar{h}_0 \equiv h_1\, \bar{r}_1 + h_2\, \bar{r}_2, \tilde{r}_0 \equiv h_1\, r_1 + h_2\, r_2.$$

From (16) we get

$$\bar{\lambda}\bar{y} - \delta\bar{k} = \frac{F}{N}, (A7)$$

in which $\bar{\lambda} \equiv \lambda + \xi$ and $\delta \equiv 1 - \delta_k$. Substitute (3) and (A3) into (A7)

$$\left(\bar{\lambda} + \bar{\lambda}\, r - \delta\right)\bar{k} + \bar{\lambda}\, h_0\, w = \frac{w\,\bar{N}}{N\,\beta}. (A8)$$

Substitute (A6) into (A8)

$$\left(\frac{(\bar{\lambda}+\bar{\lambda}\, r-\delta)\beta}{w}+\tilde{r}_0\right)\bar{k}+\frac{\tilde{r}\,\bar{k}+\bar{h}_0}{\widetilde{w}}+\beta\,\bar{\lambda}\,h_0-\chi=0.\,(A9)$$

From $\widetilde{w}=\bar{k}+hw$ and (A9), we get

$$\bar{k}^2+\widetilde{m}_1\,\bar{k}+\widetilde{m}_2=0,\,(A10)$$

where

$$\widetilde{m}_1(z)\equiv\frac{(\bar{\lambda}+\bar{\lambda}r-\delta)h\,\beta+\tilde{r}_0\,h\,w+\beta\,\bar{\lambda}\,h_0-\chi+\tilde{r}}{\widetilde{m}},\widetilde{m}_2(z)\equiv\frac{\bar{h}_0+(\beta\,\bar{\lambda}\,h_0-\chi)h\,w}{\widetilde{m}},$$

$$\widetilde{m}(z)\equiv\frac{(\bar{\lambda}+\bar{\lambda}r-\delta)\beta}{w}+\tilde{r}_0.$$

We solve (A10)

$$\bar{k}(z,t)=\frac{-\widetilde{m}_1\pm\sqrt{\widetilde{m}_1^2-4\,\widetilde{m}_2}}{2}.\,(A11)$$

We confirmed the procedure in the Lemma. From the procedure and (14), we have

$$\dot{N}(t)=\tilde{\Omega}_N(z,N,t).\,(A12)$$

Equation (15) implies

$$\dot{\bar{k}}=\Omega_0(z,t)\equiv\lambda\bar{y}-\bar{k}.\,(A13)$$

Insert $\bar{k}(t)$ from (A11) in (A13)

$$\dot{z}=\tilde{\Omega}_z(z,N,t)\equiv\left(\Omega_0-\frac{\partial\bar{k}}{\partial t}\right)\left(\frac{\partial\bar{k}}{\partial z}\right)^{-1}.\,(A14)$$

We confirmed the Lemma.

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