

# Nonlinear Sociophysics, Quantum Sociology, and Multiply Connected Topological Economics



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## Article History

Received: 9 November 2022

Revised: 13 January 2023

Accepted: 19 January 2023

Published: 24 January 2023

## How to Cite

Yi-Fang Chang. (2023). Nonlinear Sociophysics, Quantum Sociology, and Multiply Connected Topological Economics. *Sumerianz Journal of Social Science*, 6(1): 1-10.

## Abstract

First, nonlinear sociophysics and the nonlinear whole sociology are researched, which have chaos, fractal, etc. Second, quantum sociology is discussed. We propose that its bases are the extensive quantum theory and the social individual-wave duality. Third, we research the double solutions of some nonlinear equations with soliton and chaos, and the possible social meaning of chaos. On the one hand, chaos may correspond to the spread of ideas, the popularization of information, etc. On the other hand, it corresponds to the economic crisis and various social crises, etc. Fourth, chaos in corruption is discussed by mathematics. Fifth, we propose the multiply connected topological economics, in which the confidence relations and the influence functions represent various interacting strengths of different families, cliques, and systems of organization. This has a fractal structure. We propose the binary periods of the political economy by the complex function and the elliptic functions. Various applications of the mathematical and physical method are the important developing direction of modern social sciences.

**Keywords:** Sociophysics, Nonlinearity, Quantum, Chaos, Social crises, Topology, Economics.

## 1. Introduction

Sociophysics is the application of the concepts of physics to the social sciences. First, Auguste Comte (1798-1857) discussed social statics and social dynamics. Stewart (2001), suggested principles of social physics and summarized the development of social physics. Galam *et al.* (1982) introduced the models of sociophysics as a new approach to sociological collective behavior and discussed its developments (Galam, 2004;2008).

Bernard and Killworth (1997), searched social physics for social network knowledge and theory. Urry (2004), explored the increasing overlaps between sociology and physics and explained the so-called small world phenomenon and corresponding new social physics. Warntz (2005), discussed transportation, social physics, and the law of refraction. We discussed generally the four variables and the eight aspects of social physics and searched social thermodynamics and the five fundamental laws of social complex systems. Then we researched different relations among social elements and discussed the evolutionary equation of the system and the educational equation, etc (Chang, 2013a).

Sen and Chakrabarti (2014), published the book *Sociophysics: An Introduction*. Schweitzer (2018) proposed sociophysics as a physicist's modeling of psycho-political phenomena. In this paper, we research the nonlinear sociophysics, quantum sociology, and possible social meaning of chaos, and propose the multiply connected topological economics.

## 2. Nonlinear Sociophysics

Humanity as an inseparable whole on Earth possesses a common environment and benefits. Based on the inseparability and correlativity of the social systems, we proposed the nonlinear whole sociology and the four basic laws (Chang, 2013b): I. The inseparability exists always among different organizations, structures, functions, and levels within various social systems, which determines the holism of the social systems. II. Many main characteristics such as self-organization and self-adjustment of social systems must be interaction and nonlinearity.

III. From human, community, city to nation and country, various social systems of different levels possess totality and nonlinearity, whose diversity and complexity originate from various nonlinear interactions. IV. A basic property of any social system as an open system is that this system and its environment (for example, nature, geography, polity, culture, etc., and other social systems) must be a whole. Usually, the environment is regarded as a boundary condition, but it and the social systems have often various nonlinear relations.

Generally, we discuss nonlinear sociophysics, which has chaos and fractal, etc. Its nonlinear differential equations for the evolutionary social systems may be two simplified forms: 1. The equation of second order:

$$x'' + \alpha x' + f(x) = 0, \tag{1}$$

where  $f(x)$  includes some nonlinear terms, e.g.,  $bx^3$  etc. 2. A set of coupled equations of two elements:

$$\begin{aligned} x' &= a_1x + b_1y + F_1(x, y), \\ y' &= a_2x + b_2y + F_2(x, y). \end{aligned} \tag{2}$$

Here  $F(x,y)$  are some coupled terms on  $x$  and  $y$ . The nonlinear sociophysics includes the social field, whose function for the average field depends on the social energy  $E$ , the phase space  $S$ , and time  $t$ :

$$\bar{y} = \bar{f}(E_k, S_j, t) \tag{3}$$

The social field changes to pass through a critical threshold value, which may produce different structures of phase transition. This is a unification of necessity and fortuity. The threshold value at a critical point may pass through fluctuation to obtain a bifurcation point even chaos, which can produce different results of phase transition.

In modern and postmodern sociological theory (Ritzer, 1997; Ritzer and Goodman, 2004), systems theory, network theory, and globalization theory are whole theories, while structural functionalism, neo-functionalism, conflict theory, structuralism, poststructuralism, existentialism, and symbolic interactionism, etc., are inevitably nonlinear theories. The totality and the nonlinearity are two basic social characters, and both are closely related. Because of the complexity, inseparability, and correlativity of the social systems, their description must apply the nonlinear theory with the interaction terms. Reversibly, if there is no totality, any society cannot be formed. If there is no nonlinear interaction, the system cannot form a social structure.

### 3. Quantum Sociology

The first application of quantum in sociology is the Bohr principle of complementarity. Zohar and Marshall (1994), researched the quantum society from the mind, and physics as a new social vision. Wendt (2015), studied systematically quantum mind and social science and proposed that people are the walking wave functions based on the causal closure (or completeness) of physics (CCP). This research core is that quantum theory can explain the phenomena of consciousness and intentionality, and an important basis is a quantum entanglement and macroscopic entanglement.

It is known that quantum has non-locality and holism. Human and social life are shown the quantum coherence (Haven and Khrennikov, 2013), i.e., the wave function, which can be maintained by the quantum brain theory at the macroscopic and the whole-organism level (Penrose, 1994). Further, quantum psychology has progressed to the entire social sciences (Haven and Khrennikov, 2013; Heelan, 1995; Khrennikova et al., 2014).

But, some applications of quantum such as sociology have the scale question. We proposed the extensive quantum theory since 1989 (Chang, 1990;2002), which is applications in many aspects (Chang, 2014;2018). In quantum sociology, the basis is that people are an independent “quantum”, which corresponds to Leibnizian monads. Further, various social structures, organizations, and countries at all levels are different “quantum”, which have fractal with self-similarity, such as for larger and smaller countries. The ‘quantum’ entanglement in the system determines the holism of the system.

In social synergetics, we proposed the four basic theorems of social systems (Chang, 2000;2013a): 1. Theorem of outside selection and inside evolution. 2. Theorem of the golden section on social synergy. 3. Theorem of perfect correlation on humanity. It points out that everyone in the social system correlates with each other by the formula. We introduced a fight formula like a Bell inequation:

$$\frac{1}{k} [(ab) + |a - b|] = J < 1, \tag{4}$$

in which 1 is the initial state of the whole, and  $a$  and  $b$  are its parts. Because  $k$  is a fighting strength, in this case,  $J$  is always less than 1. By mathematical extremal theorem, when  $k=1$  and  $a=b=1/2$ , there is the minimum  $J=1/4$ . Further development can be in both forms:

1) For  $a - b$  is not the absolute value, so

$$\frac{1}{k} [(ab) + (a - b)] = J_1 \tag{5}$$

It shows that if fewness fights the majority, a result will become to the opposite direction, and  $J_1$  may be negative. When  $a = (3 - \sqrt{5})/2 = 0.382$ ,  $b = (\sqrt{5} - 1)/2 = 0.618$  (a golden section value),  $J_1 = 0$ . While  $a < 0.382$ ,  $J_1 < 0$ . This may explain why any despotism society of rule as fewness will tend necessarily to perdition. The Solon reformation in old Green and the imperial examinations in old China are all to enlarge the ruling group.

2) When the subsystems cooperate each other:

$$\frac{1}{k'}[(ab) + (a + b)] = J_2 \tag{6}$$

In this case, the cooperation-coefficient  $k'$  is a direct ratio with  $J_2$ . When  $a=b=1/2$  and  $k=1$ ,  $J_2=5/4$  is the maximum. Since the most increase is only  $1/4$ , but the least decrease may reach  $3/4$ , it can show quantitatively that any object is destroyed easily and is constructed hard.

Humanity as an inseparable whole on Earth possesses a common environment and benefits. A responsible idea for humanity should be set up since every behavior of humanity correlates closely. This is an extensive quantum entanglement.

4. Theorem of transformation from quantity to quality on social development. It should be a director of social development in 21 century. We must discard single linear expansion, and a developing aim should transform into multi-levels, high quality, and variety for human overall development.

Based on the social structure we proposed the social individual-wave duality, which describes the social changes with a tendency after a tendency (Chang, 2015). We researched the social strain field, the pattern dynamics, and the damage mechanics in sociology, which describe some social phenomena and the crises of society (Chang, 2017).

#### 4. Double Solution of Some Nonlinear Equations, and Social Meaning of Chaos

In nonlinear equations, the usual soliton and chaos are completely different. We proposed the double solutions of some nonlinear equations with soliton and chaos (Chang, 2013d), in which all nonlinear equations with a soliton solution may derive chaos. While only some equations with a chaos solution have a soliton. The conditions of the two solutions are different. When some parameters are certain constants, the soliton is derived; while these parameters vary in a certain region, the bifurcation-chaos appears. The former corresponds to a stable state, and the latter is a changeable process.

We search the nonlinear quantum sociology. Schrödinger equation is very important in quantum mechanics. The nonlinear Schrödinger equation is:

$$\phi_{xx} + i\phi_t + k|\phi|^2\phi = 0 \tag{7}$$

It has a soliton solution (Scott et al., 1973):

$$\phi_s = \phi_0 \operatorname{sech}\left[\sqrt{\frac{k}{2}}\phi_0(x - u_e t)\right] \exp\left[i\left(\frac{u_e}{2}\right)(x - u_e t)\right] \tag{8}$$

where the variable  $\eta = x - u_e t$ . Let

$$\phi = \exp\left[i\frac{u_e}{2}(x - u_e t)\right]v \tag{9}$$

equation (7) may become

$$\frac{dv}{d\eta} = [C + av^2 - \frac{k}{2}v^4]^{1/2} \tag{10}$$

where  $a = (u_e / 2)^2 - (u_e u_c / 2)$ . When  $C=0$ , the soliton solution is

$$v = \sqrt{\frac{2a}{k}} \operatorname{sech}^{1/2} \eta \tag{11}$$

Mathematically, a soliton may describe “extensive quantum”.

From Eq.(11) let  $v = \sqrt{2a/k} \sin x$ , the equation is

$$x' = \sqrt{a} \sin x \tag{12}$$

which has the chaos solution. For a stable state whose energy is  $H$ , if  $k=-b<0$ , the equation will be

$$\phi'' + H\phi - b\phi^3 = 0 \tag{13}$$

whose integral is

$$\phi' = (C - H\phi^2 + \frac{b}{2}\phi^4)^{1/2} \tag{14}$$

Let  $C = H^2 / 2b$ , so

$$\phi' = \sqrt{\frac{b}{2}}\left(\frac{H}{b} - \phi^2\right) \tag{15}$$

When  $|\phi| < \sqrt{H/b}$ ,

$$\phi_s = \sqrt{\frac{H}{b}} \operatorname{th}\left(\sqrt{\frac{b}{2}}\eta + C_0\right) \quad (16)$$

It is the simplest soliton with a bell shape. Using a substitution  $\phi = Hx / \sqrt{2b}$  for Eq.(15), it becomes a different equation

$$X_{n+1} = 1 - \frac{H}{2} X_n^2 \quad (17)$$

It is a known equation, which has the chaos solution, and its parameter determined the bifurcation-chaos  $\mu = H / 2$ .

Dirac equation and Klein-Gordon equation correspond to relativistic fermion and boson. Their nonlinear equations all have chaos solutions. The nonlinear Dirac equation is

$$\gamma_\mu \partial_\mu \psi + m\psi - l_0^2 \psi(\psi^+ \psi) = 0 \quad (18)$$

The probability density  $\rho = \psi^+ \psi = \bar{\psi} \gamma_4 \psi$ , and  $\psi^+ \psi = 1 - \psi \psi^+$ , so

$$\frac{\partial \rho}{\partial x_\mu} = 2\gamma_\mu l_0^2 (\psi^+ \psi)(\psi^+ \psi) - \gamma_\mu l_0^2 (\psi^+ \psi) = \gamma_\mu l_0^2 \rho(2\rho - 1) \quad (19)$$

Let  $\eta = \alpha(\gamma_\alpha x_\alpha - u\gamma_0 t)$ , the equation is

$$\frac{d\rho}{d\eta} = l_0^2 \rho(2\rho - 1) \quad (20)$$

whose solution is

$$\rho = \frac{1}{2 - \exp(-l_0^2 \eta + c)} \quad (21)$$

It is analogous to a soliton since  $\rho = (2 - e^c)^{-1} (\eta = 0)$   $\rho = 1/2 (\eta \rightarrow \infty)$  and.

The economic system is a particular social system and should be whole and nonlinear. We applied the simple logistic equation:

$$d\psi / dt = aE\psi - \psi^2 \quad (22)$$

It is the same with Eq.(20). The solution of Eq.(22) is:

$$\psi = \frac{aE}{1 + c \exp(-aEt)} \quad (23)$$

This corresponds to the increased limit in the economics of natural resources, etc., and is related to the limits to growth (Mirowski, 1989). Based on Eq.(23) and mathematical economics (Fuente, 2000), we think that the nonlinear evolution is a universal rule for economic growth, and proposed the nonlinear theory of economic growth and its three laws (Chang, 2013b): economic takeoff-growth-stagnancy law, social conservation, and economic decay law, economic growth mode transition and newly developed period law. The second law expresses a seasonal recession and two-bifurcation phenomenon of a nonlinear system. The third law connects to "the quality ladder" (Solow, 2000), which expresses a new period of development. The social open is a necessary condition for economic further development, and it must add corresponding social reform as a sufficient condition of economic development. Rivera-Batiz discussed the relationship between democracy, governance, and economic growth (Rivera-Batiz, 2002).

Further, by using the quantum mind, Wendt (2015) tries to unify the physical and social ontology. It will be possibly a new paradigm of the social sciences. We may develop quantum parapsychology.

Sociology investigates the world in change (Giddens, 2006). In society there have always been exchanges, interactions, and conflicts, then society is evolutionary (Turner, 2003). Huge conflict will derive various crises, which include economic crises, social crisis, ecological crises, resource crisis, cultural and moral crisis (Carter, 2006), political crises, etc., even if war crisis. They are some catastrophic ruptures, huge changes in structures, and collapses of systems, etc. The pattern dynamics (Curran et al., 1987) may obtain broken, rupture, bifurcation, chaos, etc. It is a process of nonlinear dynamics from uniform damage, and damage localization cumulates to sudden catastrophe and total social breakdown.

Chaos as random behavior arising in a deterministic system is common for various nonlinear social systems. We discussed social chaos and social fractals and so on (Chang, 2018). In the nonlinear equations of the social field, assume that soliton shows the orbital energy is invariant, fractal shows a self-similarity of various social systems, chaos should be related to the disorder, phase transformation, revolution, civil violence, etc.

Various diffusions are some basic characteristics of society. The enlargement of the social damage model is usually not the normal diffusion which increases linearly with time. Therefore, it is anomalous diffusion, and obeys the Brownian movement and the fractional diffusion equation:

$$\frac{\partial}{\partial t} P(x,t) = D \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \frac{\partial^2}{\partial x^2} P(x,t), \tag{24}$$

and the fractional Langevin equation:

$$m \frac{d^2 x}{dt^2} = F(x) - \gamma \Gamma(1-\alpha) \frac{d^\alpha x}{dt^\alpha} + \xi(t) \tag{25}$$

The fractional Fokker-Planck equation is:

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x} \left[ \frac{W(x)f(x,t)}{\gamma m} \right] + D \nabla^\alpha f(x,t) \tag{26}$$

These equations are simplified to form an equation of the social field. In the early days, many revolution regimes realize the average with symmetry and equality. Then the society emerges damage, which accelerates enlargement under autarchy. At a certain condition, the nonlinear system becomes bifurcation and chaos. Corruption in society is namely a type of damage.

On the one hand, chaos may correspond to the spread of ideas, the popularization of information, the formation of customs and habits, collective consciousness, and nationalism, and is consistent with the social identity theory (Schmid, 2014). This can create a relatively long-lasting society (Nakagomi, 2003). Boldrin and Montrucchio (1986), proposed that the optimum growth method is possibly nonlinear chaos.

On the other hand, it is known that chaos corresponds to the economic crisis, various social crises, and even corruption, war, etc. In the economy, an infinite clone of the same developed mode will derive a disorder in competition and finally reach necessarily to chaos and economic crisis. Therefore, nonlinear chaos economics is possibly related to crisis economics.

The social crises include impossible control of inflation to economical breakdown, social convulsions derived from corruption and huge polarization, war breaking out, etc. Crisis corresponds to chaos, which shows the complexity of sociology, is useful for the explanation of social crises, and casts a fatal shadow for an exact prediction of social crises.

Baker (1993), discussed chaos, order, sociological theory, and attractors as periphery. Stewart (2001), searched complexity theories, social theory, and social complexity. Beck (1992;1998) proposed risk society and world risk society. Jessop (2000), discussed the crisis of the national Spatio-temporal fix and the tendential ecological dominance of globalizing capitalism. Davis (2000), discussed the crises of cities. The mathematical and physical mechanisms and methods in social science may describe various social crises. We should avoid and check the sources of various social crises, and decrease the diffusion velocity of crises (Chang, 2017).

Based on the social field and social dynamics, we discussed social justice and various defects as social crises. The social hydrodynamics and corresponding waves and tides may describe positive social progress and various negative social crises from the mathematical and physical mechanisms and methods (Chang, 2017).

Social progress and crises are related to revolutions (Goldstone, 1982; Peceny, 2007). When the accumulated value of some variables once reaches a certainty, the revolution will happen possibly (Paulson, 2006).

## 5. Chaos in Corruption

In any society, many items are unchanged, at least for a certain period, for instance, matter and money, etc. Their conservation equations are:

$$\frac{\partial m}{\partial t} + \text{div}(mv) = 0 \tag{27}$$

If the divergences of items are bigger and let  $\text{div}(mv) = -1 + \lambda m^2$ , the equation (27) will become the nonlinear form:

$$\frac{dm}{dt} = 1 - \lambda m^2 \tag{28}$$

its solution is:

$$m = \text{cth}(\lambda t) / \sqrt{\lambda} \tag{29}$$

The social matter and money, etc., will decrease along with time. In this case, Eq.(28) becomes a difference equation:

$$X_{n+1} = 1 - \lambda X_n^2 \tag{30}$$

Assuming that the replacement may represent corruption from one level to another level or continuous expansive corruption, this equation may be called a corruption equation. For this, if corruption cannot be controlled effectively, the bifurcation with two periods will appear, and finally, the solution will achieve chaos, namely, the whole social system will decompose and become confused. From a quantitative value of bifurcation-chaos, we may estimate that if the corruption members in any social system or a certain level attain 37.5%, a dangerous double periodic bifurcation will appear. If the members attain 0.70057759...=70%, this system as a whole will show an incurable dark. It is some quantitative threshold values for a social system into corruption, and represented visually

in a known figure from bifurcation to chaos (Fig.1). This has also fractal character no matter what big or small system, and big or small country.

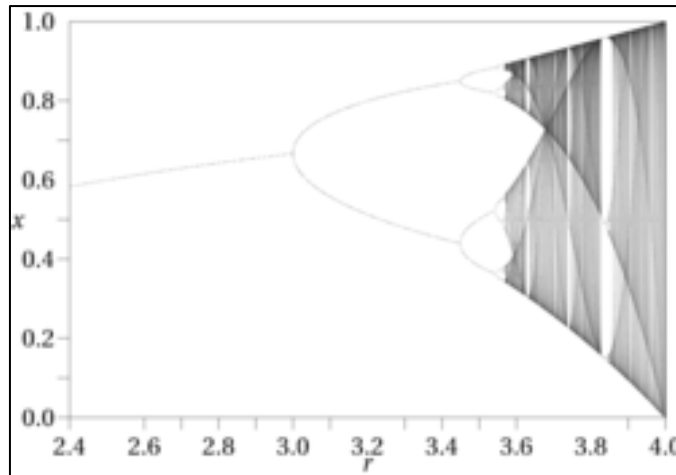


Fig-1. Bifurcation-chaos diagram

## 6. Multiply Connected Topological Economics

Topological economics is known. In mathematical economics, the fixed-point theorems of topology are used to prove the Nash equilibrium for n-person games. Arrow and Debreu (1954) presented a general model of Walrasian equilibrium theory and proved the existence theorem of equilibrium for a competitive economy by topology. Then McKenzie (1959), Debreu (1962) developed the competitive equilibrium theory. Differential topology is introduced into economics, and Debreu discussed two detailed questions.

Mirowski (1989), discussed economics as social physics. Mantegna and Stanley (2000), introduced econophysics as correlations and complexity in finance. Chatterjee et al. (2005), edited the book *Econophysics of Wealth Distributions*. Kutnera et al. (2019), searched econophysics and sociophysics, and their milestones and challenges.

In the microeconomic theory of consumer behavior, either a utility function or a binary relation can describe the preferences of an individual. The strict equivalence of these two primitive concepts, ordinal utility functions, and preference relations, was first axiomatized by Debreu (1959;1964). He studies the concept of cardinal utility in three different situations using the same mathematical result that gives a topological characterization of three families of parallel straight lines in a plane, and discussed that for every continuous complete and transitive binary relation  $\geq$  defined on an arbitrary subset X of the commodity space R, there is a continuous utility representation; that is, there is a continuous function u of X into R such that  $u(y) \geq u(x)$  if and only if  $y \geq x$ . Therefore, the more basic concept of preferences is applied instead of utility using a topology or a metric on the space of preferences. Undoubtedly, it is a great contribution to economics.

In microeconomics, we introduced the confidence relations that represent various interacting strengths of different families, cliques, and systems of organization. It is an important human relationship in economics, even independent of economic results. The confidence relation can be defined by a similar method to the preference relation in consumer theory (Chang, 2013c).

The confidence relation  $\geq$  defined on the choice set X is a complete preordering, continuous, and strictly monotone. This requires:

- (1). Reflexivity:  $\forall x \in X, x \geq x$
- (2). Completeness:  $\forall x, y \in X$  either  $x \geq y$  or  $y \geq x$  both;
- (3). Transitivity:  $\forall x, y, z \in X, [x \geq y \text{ and } y \geq z] \Rightarrow x \geq z$ .

The  $\geq$  can be represented by a real-valued, continuous, and increasing payoff function.

Further, the definition of the influence function I am similar to the utility function: A real-valued function  $I^i : X^i \rightarrow R$  represents confidence preordering  $\{\geq_i\}$  defined on the choice set  $X^i$  of agent I if  $\forall x, y \in X^i, x \geq_i y \Leftrightarrow I^i(x) \geq I^i(y)$ . The influence function that represents a confidence preorder is not uniquely defined. Any monotonically increasing transformation  $\phi(\cdot)$  of  $I(\cdot)$  will represent the same confidences, because with  $\phi(\cdot)$  strictly increasing, we have

$$I(x) \geq I(y) \text{ if and only if } \phi[I(x)] \geq \phi[I(y)], \quad (31)$$

for all  $\forall x, y \in X$ . Hence  $I(\cdot)$  is an ordinal influence function. The sign of the difference  $I(x) - I(y)$  is important because it tells us which outcome is confided, but the value of this difference is meaningless, as it will change with any nontrivial increasing transformation  $\phi(\cdot)$ . It is also a basic characteristic of topology, where those

concrete spacing values are meaningless. Although the influence function is similar to the utility function that obeys the law of diminishing marginal utility, the influence function seems to obey the law of augmenting lust for power.

The confidence relation, the corresponding influence function  $I(\cdot)$ , and the function  $\varphi(\cdot)$  can affect products  $Q$ , profit and prices, etc., in an economic system. But, they are usually independent of economic results and sometimes are stochastic, even changing suddenly. In a continuous topological manifold of economics, they break easily original structure and form a new hole or branch region. This will construct a multiply connected topological manifold. In an image, the economic structure is a cup, while the influence function is a handle.

In a multiply connected region of topology, there is a famous Euler-Poincare formula

$$\sum_{m=1}^n (-1)^m a_m = \sum_{m=1}^n (-1)^m p_m \quad (32)$$

For a convex polyhedron,  $a_0, a_1, a_2$  denote the number of vertices, edges, and faces, respectively;  $P^m$  is the  $m$ th Betti number of complex  $K$ . This may be considered intuitively as the numbers of  $m$ -dimensional holes in  $K$ , or is the number of  $(m+1)$ -dimensional chains that must be added to  $K$  so that every free  $m$ -cycle on  $K$  is a boundary

(Diestel, 2000). The number  $\sum_{m=1}^n (-1)^m a_m$  is called the Euler characteristic of the complex  $K$ . In the polyhedron  $p_0 = p_2 = 1, p_1 = 2p$ ,  $p$  is the deficiency of a curved surface. In a 2-dimensional curved surface,  $a_0 = a_1 + 1 - 2p$ . Assume that vertices represent the number of markets, which is directly proportional to the sales volume  $y$  and the profit, and edges represent the market network. But the multiply connected economy brings the profit decrease. In this case, there is a defective profit due to the deficiency  $p$ .

In some systems of organization, profit maximization and confidence relations are inseparable. The aim of a pure producer is the profit maximization

$$\pi(y, w) = \max_{x,y} \{py - wx\} \quad (33)$$

where  $y$  and  $x$  are output and input, and  $p$  and  $w$  are output and input prices. For a social system with the influence function  $I$ , we should define an aim function as

$$A = \pi + I \quad (34)$$

When the changes of the product and the influence are independent of one another, they may be a node or saddle point. Various powers produce usually economic wormholes and various corruptions. It is a mathematical application to economics. The form of the influence function can be an unrestricted function, even a stochastic function. Perfect competition prevails that each producer and consumer regards the prices paid and received as independent of his own choices (Arrow and Debreu, 1954). An economy with confidence relations and influence functions is a type of imperfect competitive economic system and breaks the symmetries in economic topology. They are not homeomorphic spaces. Usually, this structure will hinder economic development. If the confidence

relations and the influence functions have  $p$ -levels or  $p$ -types, i.e.  $\sum_{i=1}^p I_i(Q)$ , they will construct a multiply connected normal curved surface with the deficiency  $p$ . When the influence function is large enough to achieve a certain threshold value, the economic elasticity of the topological structure will be broken, and a new hole will appear. The unified market economy will be riddled with holes. This will form a new multiply connected topological manifold. Using the concept of general relativity a large influence (as mass) forms a pit in the economic system. A very strong pit can construct a wormhole, sometimes called the Einstein-Rosen bridge based on the nonlinear equations of general relativity. Therefore, some capital will pass through a throat into another topological space, or from a region to another region in the same space. This model will describe a loss of capital (including waste, and corruption).

A complex world may be described by the complex functions, which obtain often multivalued. For instance, a function of a single value  $e^{inx}$  is a logarithmic spiral curve and corresponds to exponential economic growth. It develops into a complex function:

$$e^{inx} = \cos nx + i \sin nx \quad (35)$$

This is a multivalued function and corresponds to periodic economic growth. A complex equation  $p(\alpha, y) = 0$  is  $n$ -th order of  $y$ , for a plane of  $x = \alpha$ , there are the  $n$ -layers curved surfaces of  $y$ .

If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. The political economy is usually an imperfect economic question, even completely not an economic question for some particular cases. It is not a strict economic rule, because in this case economy is only an appendage of the polity. The economy will change along with polity.

The political economy as a multiply connected topological economics can be described by the complex function and the elliptic functions, which have two periods of economy and policy. The complex function corresponds to a complex social system of economy and polity. This is a nonlinear whole economy. In the multiply connected topological economy, economy corresponds to a real part, and policy and relation, etc., correspond to an imaginary

part. A complex plane corresponds to the surface of the Riemann sphere, which is called a stereographic projection (Brown and Churchill, 2009).

The Cauchy-Goursat theorem (Brown and Churchill, 2009) is the closed contour to simply connected domains

$$\oint_C f(z)dz = 0 \tag{36}$$

For multiply connected domains

$$\oint_C f(z)dz = \sum_{k=1}^z \oint_{C_k} f(z)dz \tag{37}$$

An elliptic function is a meromorphic function on the complex plane (Lang, 1987), and has double periods:

$$f(z + m\omega + n\omega') = f(z) \tag{38}$$

Here  $\omega$   $\omega'$  are the two different basic periods. In this case, economical development will possess two periods of economy and policy, respectively. It includes the Weierstrass elliptic function:

$$W(z) = \frac{1}{z^2} + \sum_{\omega \in L} \left[ \frac{1}{(z-\omega)^2} - \frac{1}{\omega^2} \right] \tag{39}$$

where the sum is taken over the set of all non-zero periods, denoted by Lang (1987). Its expansion shows different influence degrees of economy or policy, for example, the confidence relation for different families, cliques, and systems of organization (Chang, 2013c). There is a theorem (Lang, 1987): Assume that the elliptic function  $f$  has no poles on its boundary, then the sum of the residues of  $f$  is 0, i.e.,

$$2\pi i \sum \text{Res } f = \int_{\partial P} f(z)dz = 0 \tag{40}$$

It may describe a special case: Economy and policy cancel out, so the economy cannot be developed. Its corollary is: An elliptic function has at least two poles on the torus, i.e., the two centers of economy and policy exist simultaneously. The elliptic curves with singular invariants have complex multiplication from an imaginary quadratic field.

The Weierstrass zeta function is (Lang, 1987):

$$\zeta(z, L) = \zeta(z) = \frac{1}{z} + \sum_{\omega \in L} \left[ -\frac{1}{z-\omega} + \frac{1}{\omega} + \frac{z}{\omega^2} \right] \tag{41}$$

There have  $\zeta' = -W(z)$  and

$$\zeta(\lambda z, \lambda L) = \frac{1}{\lambda} \zeta(z, L) \tag{42}$$

In this case, it is a scaling invariance and has a fractal. There is a constant  $\eta(\omega)$  such that

$$\zeta(z + \omega) = \zeta(z) + \eta(\omega) \tag{43}$$

Their separation is namely a connected graph becomes a non-connected graph (Bollobas, 2002; Diestel, 2000); in graph theory of the political economy.

The multiply connected topological economics has a social fractal, which describes that different systems possess self-similarity from the company, society to country. Their sizes are different, while structures are similar, and possess the same function. In this case, the mathematics may be applied to the scale theory and the renormalization group.

In Chinese traditional society, the periodic changes of dynasties show the two periods of economy and policy, and both influence each other. The two periods of economics and corruption also influenced each other.

When  $\alpha \in M_2^+(\mathbb{Z})$  (the sets of  $2 \times 2$  matrix with components in  $\mathbb{Z}$ ) and  $\det \alpha = N$ , then  $j \circ \alpha$  is a modular function of level  $N$  (Lang, 1987). Here  $\alpha$  is the matrix, so it may correspond to the input-output model, and two aspects of economy and policy. It may have higher levels, which include economy and policy. In the elliptic function, there are subfields of the modular function field. In the multiply connected topological economy, economy and policy are different subfields.

The multiply connected topological economy may be extended to various relations between the economy and other politics, family, religion, etc. Further, it may be developed in many regions without direct relations with the economy, such as welfare, environment, full employment, etc.

## 7. Conclusion

An important developing direction of modern social sciences is the wide application of various mathematical and physical methods. A particular example is quantum theory.

Consciousness is a macroscopic quantum theory (Wendt, 2015). Panpsychism shows that except matter-energy-information, the world must add mind, psi, etc. This not only corresponds to dualism, and is both aspects of unification and neutral monism, i.e., Chinese traditional Lu-Wang Mind Philosophy (mind=matter).



Nonlinearity is a basic character of many social systems, which have often fractal structures. When the order parameter reaches a certain value, chaos appears, so the economy collapses, war breaks out and various social crises appear.

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